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Judul Publikasi A new orthogonality and angle in a normed space

by

Submission date: 28-Nov-2021 07:47PM (UTC+0700)
Submission ID: 1714096614
File name: A_new_orthogonality_and_angle_in_a_normed_space.pdf (428.38K)
Word count: 3442
Character count: 14353

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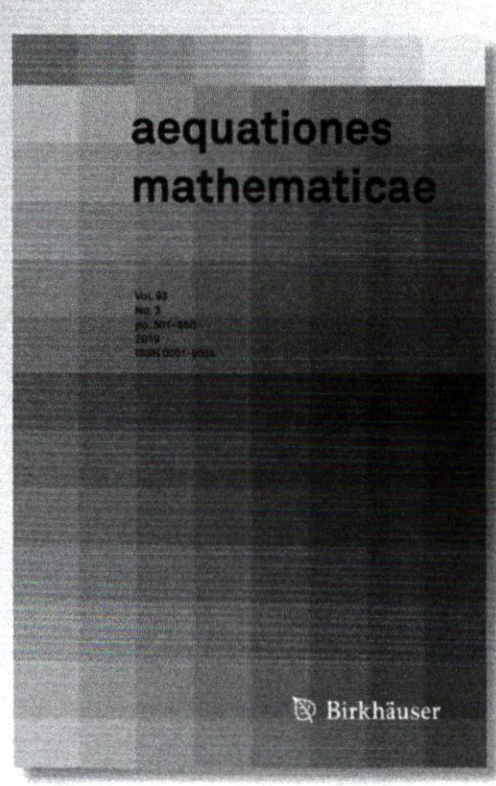
Aequationes mathematicae

ISSN 0001-9054
Volume 93
Number 3

Aequat. Math. (2019) 93:547-555
DOI 10.1007/s00010-018-0582-3



Springer

Aequat. Math. 93 (2019), 547–555

0001-9054/19/030547-9
published online July 14, 2018
https://doi.org/10.1007/s00010-018-0582-3

Aequationes Mathematicae

A new orthogonality and angle in a normed space

M. Nur and H. Gunawan

Abstract. We introduce the notion of *gg*-orthogonality in a normed space and discuss its basic properties. We also show the connection between *gg*-orthogonality and *g*-orthogonality introduced by Miličić (Mat Vesnik 39:325–334, 1987). Using *gg*-orthogonality, we introduce the notion of *gg*-angle between two vectors in a normed space and discuss its properties. Moreover, we apply the *gg*-angle to examine whether or not a normed space is strictly convex.

Mathematics Subject Classification. 15A03, 46B20, 51N15.

Keywords. *gg*-orthogonality, *gg*-angle, Normed spaces, Strictly convex.

1. Introduction

In an inner product space $(X, \langle \cdot, \cdot \rangle)$, the concept of orthogonality plays important roles related to the concept of projection, orthonormality, approximation, and angles between two vectors. Two vectors x and y in X are said to be orthogonal (denoted $x \perp y$) if and only if $\langle x, y \rangle = 0$. One may observe that this orthogonality satisfies the following basic properties (see [1]):

(a) *Nondegeneracy* If $x \perp x$, then $x = 0$.
(b) *Symmetry* If $x \perp y$, then $y \perp x$.
(c) *Homogeneity* If $x \perp y$, then $\alpha x \perp \beta y$ for every $\alpha, \beta \in \mathbb{R}$.
(d) *Additivity* If $x \perp y$ and $x \perp z$, then $x \perp (y + z)$.
(e) *Resolvability* For every $x, y \in X$ there exists $\alpha \in \mathbb{R}$ such that $x \perp (\alpha x + y)$.
(f) *Continuity* If $x_n \to x$ and $y_n \to y$ (in the induced norm) and $x_n \perp y_n$ for every $n \in \mathbb{N}$, then $x \perp y$.

Now suppose that $(X, \|\cdot\|)$ is a normed space. As it is known, not all normed spaces are inner product spaces. In a normed space, several notions of orthogonality have been introduced by many authors—see, for example, [1,2,5,6,9,12,13].

Let $(X, \|\cdot\|)$ be a real normed space. We define a functional $g : X^2 \to \mathbb{R}$ by the formula

$$g(x,y) := \frac{1}{2}\|x\| \left[\tau_+(x,y) + \tau_-(x,y)\right],$$

with

$$\tau_\pm(x,y) := \lim_{t\to\pm 0} \frac{\|x+ty\| - \|x\|}{t}.$$

The following properties of the functional g, which will be useful in this paper, can be found in e.g. [3,9].

(1) $g(x,x) = \|x\|^2$ for every $x \in X$;
(2) $g(ax, by) = ab \cdot g(x,y)$ for every $x, y \in X$ and $a, b \in \mathbb{R}$;
(3) $g(x, x+y) = \|x\|^2 + g(x,y)$ for every $x, y \in X$;
(4) $|g(x,y)| \le \|x\| \cdot \|y\|$ for every $x, y \in X$;
(5) For all $x \in X$, the mapping $g(x,\cdot)$ is continuous.

If, in addition, the functional $g(x,\cdot)$ is linear for all x, then g is called a *semi-inner product* on X. For example, the functional

$$g(x,y) := \|x\|_p^{2-p} \sum |\xi_k|^{p-1} \operatorname{sgn}(\xi_k)\, \eta_k, \quad x := (\xi_k), y := (\eta_k) \in l^p \qquad (1)$$

is a semi-inner product on ℓ^p $(1 \le p < \infty)$ [10,11].

Using a semi-inner product g, Miličič [13,14] introduced notions of g-*orthogonality* on X, namely:

$$x \perp_g y \iff g(x,y) = 0$$
$$x \perp^g y \iff g(x,y)g(y,x) = 0$$
$$x \overset{g}{\perp} y \iff g(x,y) + g(y,x) = 0$$
$$x \underset{g}{\perp} y \iff \|x\|^2 g(x,y) + \|y\|^2 g(y,x) = 0.$$

Note that in an inner product space $(X, \langle\cdot,\cdot\rangle)$, the semi-inner product $g(x,y)$ is identical with the inner product $\langle x,y\rangle$, and $x\,\Omega\, y$ if and only if $\langle x,y\rangle = 0$ holds for every $\Omega \in \{\perp_g, \perp^g, \overset{g}{\perp}, \underset{g}{\perp}\}$.

In this paper, we will introduce a new notion of orthogonality by using a semi-inner product g and discuss its properties. We will also discuss its relation to at least one g-orthogonality. Using the new notion of orthogonality, we will define a new notion of angle between two vectors in X and discuss its properties. Moreover, we apply the new angle between two vectors to examine the strict convexity of a normed space.

2. Main results

2.1. gg-orthogonality

Let g be a semi-inner product on X, we define the mapping $[\cdot,\cdot]_{gg}$ on $X \times X$ by

$$[x,y]_{gg} = \sqrt{|g(x,y)||g(y,x)|}. \tag{2}$$

Note that in a real inner product space, $[x,y]_{gg} = |\langle x,y\rangle|$. Next, using properties of the semi-inner product g, we have the following result.

Proposition 2.1. *The mapping* (2) *satisfies the following properties:*

(a) $[x,x]_{gg} = \|x\|^2$ *for every* $x \in X$;
(b) $[x,y]_{gg} = [y,x]_{gg}$ *for every* $x,y \in X$ *and* $a,b \in \mathbb{R}$;
(c) $[ax,by]_{gg} = |ab| \cdot [x,y]_{gg}$ *for every* $x,y \in X$ *and* $a,b \in \mathbb{R}$;
(d) $[x,y]_{gg} \le \|x\| \cdot \|y\|$ *for every* $x,y \in X$.

Remark 2.2. The mapping $[\cdot,\cdot]_{gg}$ does not satisfy the triangle inequality with respect to each component. In other words: $[x,y+z]_{gg} \le [x,y]_{gg} + [x,z]_{gg}$ does not hold in general. For example, consider ℓ^1 with the semi-inner product g in (1). Take $x = (3,1,0,\ldots)$, $y = (2,-2,0,\ldots)$, and $z = (0,2,0,\ldots)$. Clearly $\|x\| = 4$, $\|y\| = 4$, $\|z\| = 2$, $\|y+z\| = 2$, $g(x,y) = 0$, $g(y,x) = 8$, $g(x,z) = 8$, $g(z,x) = 2$, $g(x,y+z) = 8$, $g(y+z,x) = 6$. Hence $[x,y+z]_{gg} > [x,y]_{gg} + [x,z]_{gg}$.

By using the mapping $[\cdot,\cdot]_{gg}$ in (2), we define a new orthogonality as follows.

Definition 2.3. (gg-*orthogonality*) We say that x is gg-*orthogonal to* y, and we write $x \perp_{gg} y$, if and only if $[x,y]_{gg} = 0$.

Note that in an inner product space $(X,\langle\cdot,\cdot\rangle)$, gg-orthogonality coincides with the usual orthogonality. As a consequence of Proposition 2.1, we have the following proposition.

Proposition 2.4. gg-*orthogonality satisfies the following properties:*

(a) Nondegeneracy: *If* $x \perp_{gg} x$, *then* $x = 0$.
(b) Symmetry: *If* $x \perp_{gg} y$, *then* $y \perp_{gg} x$.
(c) Homogeneity: *If* $x \perp_{gg} y$, *then* $\alpha x \perp_{gg} \beta y$ *for every* $\alpha,\beta \in \mathbb{R}$.
(d) Resolvability: *For every* $x,y \in X$ *there is* $\alpha \in \mathbb{R}$ *such that* $x \perp_{gg} (\alpha x + y)$.

Proof. Using Proposition 2.1, the statements (a)−(c) are obviously true.

(d) Let $x,y \in X - \{0\}$. Then for $\alpha = -\frac{g(x,y)}{\|x\|^2}$, we have

$$\begin{aligned}[x,\alpha x+y]_{gg} &= \sqrt{|g(x,\alpha x+y)||g(\alpha x+y,x)|}\\ &= \sqrt{|\alpha g(x,x)+g(x,y)||g(\alpha x+y,x)|}\\ &= 0,\end{aligned}$$

as desired. □

Remark 2.5. gg-orthogonality does not satisfy the additivity property. For instance, consider ℓ^1 with the usual semi-inner product g. Take $x = (1, 1, 0, \ldots)$, $y = (-1, 2, 0, \ldots)$, and $z = (2, -1, 0, \ldots)$. Clearly $g(x, y) = 2$, $g(y, x) = 0$, $g(x, z) = 2$, $g(z, x) = 0$, $g(x, y + z) = 4$, and $g(y + z, x) = 4$. We have $[x, y]_{gg} = 0$, $[x, z]_{gg} = 0$, and $[x, y + z]_{gg} = 4$. Hence $x \perp_{gg} y$ and $x \perp_{gg} z$, but x is not gg-orthogonal to $y + z$. Likewise, gg-orthogonality does not satisfy the continuity property. For instance, in ℓ^1 with the above semi-inner product g, take $x_n = (\frac{1}{n}, -1, 0, \ldots)$, $x = (0, 1, 0, \ldots)$, and $y = (1, 1, 0, \ldots)$. We obtain $x_n \to x$ (in the norm), $[x_n, y]_{gg} = 0$, but $[x, y]_{gg} \neq 0$.

2.2. The connection between gg-orthogonality and g-orthogonality

We shall present here the connection between gg-orthogonality and g-orthogonality presented in the introduction.

Fact 2.6. *Let g be a semi-inner product on $(X, \|\cdot\|)$.*

(a) *If $x \perp_g y$, then $x \perp_{gg} y$.*
(b) *$x \perp^g y$ if and only if $x \perp_{gg} y$.*

Remark 2.7. The converse of part (a) is not true. For example, take $x = (1, 1, 0, \ldots)$ and $y = (-1, 2, 0, \ldots)$ in ℓ^1 with the usual semi-inner product g. One may check that $[x, y]_{gg} = 0$ but $g(x, y) = 2$.

We have seen that gg-orthogonality does not satisfy the additivity and the continuity properties. However, we have the following corollary about the 'additivity' property and the 'continuity' property with respect to the second component, which may be proved by using some properties of g-orthogonality and Fact 2.6.

Corollary 2.8. *Let g be a semi-inner product on $(X, \|\cdot\|)$. Then*

(a) *If $x \perp_g y$ and $x \perp_g z$, then $x \perp_{gg} y + z$.*
(b) *If $y_n \to y$ (in the norm) and $x \perp_g y_n$ for every $n \in \mathbb{N}$, then $x \perp_{gg} y$.*

In addition, we also have results related to the 'continuity' property as follows.

Proposition 2.9. *Let g be a semi-inner product on $(X, \|\cdot\|)$. Then*

(a) *If $x_n \to x$ and $y_n \to y$ (in the norm), $x \perp_g y_n$ and $y \perp_g x_n$ for every $n \in \mathbb{N}$, then $x \perp_{gg} y$.*
(b) *If $y_n \to y$ (in the norm), $y_n \perp_{gg} x$ and $y_n \not\perp_g x$ for every $n \in \mathbb{N}$, then $x \perp_{gg} y$.*

Proof. (a) Suppose that $x \perp_g y_n$ and $y \perp_g x_n$. Observe that

$$\begin{aligned}[x,y]_{gg}^2 &= |g(x,y)||g(y,x)| \\ &= |g(x,y) - g(x,y_n)||g(y,x) - g(y,x_n)| \\ &= |g(x,y-y_n)||g(y,x-x_n)| \\ &\le \|x\|\|y\|\|x_n - x\|\|y_n - y\|.\end{aligned}$$

Because $\|x_n - x\| \to 0$ and $\|y_n - y\| \to 0$, we have $[x,y]_{gg} = 0$.

(b) Suppose that $y_n \perp_{gg} x$ and $y_n \not\perp_g x$. Using the function $[\cdot,\cdot]_{gg}$ in (2), we have $x \perp_g y_n$. Since $y_n \to y$ (in the norm), we obtain $x \perp_{gg} y$ by Corollary 2.8 part (b).

2.3. gg-angle between two vectors

In an inner product space $(X, \langle\cdot,\cdot\rangle)$, the angle $A(x,y)$ between two nonzero vectors x and y in X is given by

$$A(x,y) := \arccos \frac{\langle x,y\rangle}{\|x\|\|y\|}$$

where $\|x\| := \langle x,x\rangle^{\frac{1}{2}}$ denotes the induced norm in X. One may observe that the angle $A(x,y)$ in X satisfies the following basic properties (see [7]):

(a) *Parallelism* $A(x,y) = 0$ if and only if x and y are of the same direction; $A(x,y) = \pi$ if and only if x and y are of opposite direction.
(b) *Symmetry* $A(x,y) = A(y,x)$ for every $x, y \in X$.
(c) *Homogeneity*

$$A(ax, by) = \begin{cases} A(x,y), & ab > 0 \\ \pi - A(x,y), & ab < 0. \end{cases}$$

(d) *Continuity* If $x_n \to x$ and $y_n \to y$ (in the induced norm), then $A(x_n, y_n) \to A(x,y)$.

In a normed space $(X, \|\cdot\|)$, several notions of angle between two vectors have been studied in [4,8,11,16,17]. Using the above gg-orthogonality, we can now define the gg*-angle* between two nonzero vectors x and y in X, denoted $A_{gg}(x,y)$, by the formula

$$A_{gg}(x,y) := \arccos \frac{[x,y]_{gg}}{\|x\| \cdot \|y\|}.$$

Note that $0 \le A_{gg}(x,y) \le \frac{1}{2}\pi$ and $A_{gg}(x,y) = \frac{1}{2}\pi$ if and only if $[x,y]_{gg} = 0$ or $x \perp_{gg} y$. In an inner product space $(X, \langle\cdot,\cdot\rangle)$, $A_{gg}(x,y) = A(x,y)$ provided that $\langle x,y\rangle \ge 0$.

Example 2.10. Consider ℓ^1 with the usual semi-inner product g. Take $x = (1, 2, 0, 0, \ldots)$, $y = (-1, 3, 0, 0, \ldots)$. Clearly $\|x\| = 3$, $\|y\| = 4$, $g(x, y) = 6$, and $g(y, x) = 4$. Hence, $A_{gg}(x, y) = \arccos(\frac{1}{6}\sqrt{6})$.

Theorem 2.11. *The gg-angle $A_{gg}(\cdot,\cdot)$ satisfies the following properties:*

(a) *If x and y are linearly dependent, then $A_{gg}(x, y) = 0$ (part of the parallelism property).*
(b) *$A_{gg}(x, y) = A_{gg}(y, x)$ for every $x, y \in X$ (symmetry property).*
(c) *$A_{gg}(ax, by) = A_{gg}(x, y)$ for every $x, y \in X$ and $a, b \in \mathbb{R} - \{0\}$ (part of the homogeneity property).*

Proof. (a) Let $y = kx$ for an arbitrary nonzero vector x in X and $k \in \mathbb{R} - \{0\}$. Using Proposition 2.1, we have

$$A_{gg}(x, y) = \arccos \frac{[x, kx]_{gg}}{\|x\| \cdot \|kx\|} = \arccos \frac{|k| \cdot [x, x]_{gg}}{|k| \|x\|^2} = 0.$$

(b) Using Proposition 2.1, we have

$$A_{gg}(x, y) = \arccos \frac{[x, y]_{gg}}{\|x\| \cdot \|y\|} = \arccos \frac{[y, x]_{gg}}{\|y\| \cdot \|x\|} = A_{gg}(y, x).$$

(c) Let a and $b \in \mathbb{R} - \{0\}$. Observe that

$$A_{gg}(ax, by) = \arccos \frac{|ab| \cdot [x, y]_{gg}}{|ab|(\|x\| \cdot \|y\|)} = A_{gg}(x, y),$$

as desired. □

Remark 2.12. The converse of part (a) is not true. For instance, consider ℓ^1 with the usual semi-inner product g. Take $x = (2, 1, 0, \ldots)$ and $y = (1, 2, 0, \ldots)$. Clearly x and y are linearly independent. But one may check that $A_{gg}(x, y) = 0$. Likewise, $A_{gg}(x, y)$ does not satisfy the continuity property. For instance, in ℓ^1 with the above semi-inner product g, take $x_n = (\frac{1}{n}, 1, 0, \ldots), y_n = (1 + \frac{1}{n}, 1, 0, \ldots), x = (0, 1, 0, \ldots)$ and $y = (1, 1, 0, \ldots)$. We obtain $[x_n, y_n]_{gg} \nrightarrow [x, y]_{gg}$. Hence, $A_{gg}(x_n, y_n) \nrightarrow A_{gg}(x, y)$. From (c), we see that the gg-angle between two vectors is also the gg-angle between two lines in a normed space.

Using the definition of the g-angle between two vectors x and y, that is, $A_g(x, y) := \arccos \frac{g(x,y)}{\|x\| \cdot \|y\|}$ in [11] and the gg-angle, we have the following fact.

Fact 2.13. *Let g be a semi-inner product on $(X, \|\cdot\|)$.*

(a) *If $A_g(x, y) = \frac{\pi}{2}$, then $A_{gg}(x, y) = \frac{\pi}{2}$.*
(b) *If $A_{gg}(x, y) = 0$, then $A_g(x, y) = 0$ or $A_g(x, y) = \pi$.*

Proof. (a) If $A_g(x, y) = \frac{\pi}{2}$, then $x \perp_g y$ by [11]. According to Fact 2.8, we obtain $x \perp_{gg} y$. Hence $A_{gg}(x, y) = \frac{\pi}{2}$.

(b) If $A_{gg}(x,y)=0$, then

$$\frac{\sqrt{|g(x,y)||g(y,x)|}}{\|x\|\|y\|}=1.$$

Consequently, we have $|g(x,y)||g(y,x)|=\|x\|^2\|y\|^2$. Using the properties of the functional g, we obtain

$$|g(x,y)|=|g(y,x)|=\|x\|\,\|y\|.$$

Clearly, $\cos A_g(x,y)=\pm 1$. Hence, $A_g(x,y)=0$ or $A_g(x,y)=\pi$. □

Remark 2.14. The converse of part (a) is not true. For instance, consider ℓ^1 with the usual semi-inner product g. Take $x=(1,1,0,\ldots)$ and $y=(-1,3,0,\ldots)$, so that we have $\|x\|=2$, $\|y\|=4$, $g(x,y)=4$, and $[x,y]_{gg}=0$. Hence, $A_{gg}(x,y)=\frac{\pi}{2}$ but $A_g(x,y)=\frac{\pi}{3}$. Likewise, the converse of part (b) is not true. For instance, in ℓ^1 with the above semi-inner product g, take $x=\pm(2,1,0,\ldots)$ and $y=(1,0,0,\ldots)$. We obtain $\|x\|=3$, $\|y\|=1$, $g(x,y)=\pm 3$, and $[x,y]_{gg}=\sqrt{6}$. Hence, $\cos A_g(x,y)=\pm 1$ but $\cos A_{gg}(x,y)=\frac{1}{3}\sqrt{6}$.

3. An application

Using the gg-angle, we can examine the strict convexity of a normed space $(X,\|\cdot\|)$. First, we recall the definition of strict convexity as follows.

Definition 3.1. [10] A normed space is *strictly convex* if whenever $\|x\|+\|y\|=\|x+y\|$ where $x,y\neq 0$, we have $y=\lambda x$ for some real $\lambda>0$.

Giles [10] gives the connection between strict convexity and the semi-inner product in the following lemma.

Lemma 3.2. [10] *Let $(X,\|\cdot\|)$ be a normed space and $[\cdot,\cdot]$ be a semi-inner product. Then the following statements are equivalent:*

(1) *X is strictly convex.*
(2) *If $[x,y]=\|x\|\,\|y\|$ where $x,y\neq 0$, then $y=\lambda x$ for some real $\lambda>0$.*

Using the above lemma and the gg-angle, we obtain the following result.

Theorem 3.3. *Let g be a semi-inner product on $(X,\|\cdot\|)$ and $A_{gg}(x,y)$ be the gg-angle between two vectors x and y in X. Then the following statements are equivalent:*

(1) *X is strictly convex.*
(2) *If $\cos A_{gg}(x,y)=1$ where $x,y\neq 0$, then $y=\lambda x$ for some real $\lambda>0$.*

Proof. Suppose that (1) holds, that is, X is strictly convex. If $\cos A_{gg}(x, y) = 1$, then

$$|g(x, y)||g(y, x)| = \|x\|^2\|y\|^2.$$

Using the properties of t[redacted] functional g, we obtain

$$|g(x, y)| = |g(y, x)| = \|x\|\,\|y\|.$$

Since $g(x, y)$ is a semi-inner product on X, by Lemma 3.2 we have $y = \lambda x$ for some real $\lambda > 0$. This means that (2) holds.

Conversely, suppose [redacted] (2) holds. We shall prove (1) by employing Lemma 3.2 once again. Su[redacted]se that $g(x, y) = \|x\|\,\|y\|$ where $x, y \neq 0$. It follows from [15] that $g(y, x) = \|x\|\,\|y\|$. Hence, $\cos A_{gg}(x, y) = 1$ where [redacted] $\neq 0$. By assumption, we obtain $y = \lambda x$ for some real $\lambda > 0$. Therefore, $g(x, y) = \|x\|\,\|y\|$ (where $x, y \neq 0$) implies that $y = \lambda x$ for some real $\lambda > 0$. By Lemma 3.2, we conclude that X is strictly convex.

Acknowledgements

The research [redacted] supported by ITB Research and Innovation Program 2018. The authors thank the referee for his/her useful comments and suggestions on the earlier version of this paper.

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Received: March 2, 2018
Revised: May 25, 2018

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