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The Total Edge Irregular Labeling of Network Constructed by Some Copies of Cycle on Three Vertices Corona a Vertex

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Abstract

Many networks have been found the total edge irregularity strength's. The aim of this paper is determined the total edge irregularity strength of network constructed by some copies of cycle on three vertices corona a vertex. The results of this paper that the total edge irregularity strength of network constructed by some copies of cycle on three vertices corona a vertex is $2m+1$ for $m \geq 2$ where m is the number copies of cycle on three vertices.

Keywords: Corona Product of Graph, Irregular Labeling, Irregularity Strength

1. Introduction

A graph labeling is one of models to illustrate the approach for any network. A *graph* G consists of a finite set $V(G)$ of objects called vertices together with a set $E(G)$ of an ordered pairs of *vertices*; the elements of $E(G)$ are called *edges*. The edge e containing x and y is written $e = xy$, where x and y are called *endpoints*of e.

Formally, a *graph labeling* is a map that carries graph elements to the numbers (usually to the positive or non-negative integer). A labeling of a graph is called a *vertex labeling*, an *edge labeling*, or a *total labeling*, if the domain of this map is the vertex set, the edge set, or the union of vertex and edges set, respectively[17].

In 2002, Ba$\check{c}$a *et al*. introduced an edge irregular total *k*-labeling of any graph. A total labeling $\alpha : V \cup E \to \{1, 2, \cdots, k\}$ is called a *total edge irregular k-labeling* of G if the weight of all edges is distinct. The weight of edge $e = xy$ under the total labeling α is $\alpha(e) + \alpha(x) + \alpha(y)$, where x and y are vertices in a graph.

For some positive integer k, any graph have a total edge irregular k-labeling. The interesting is the minimum positive integer umber k for which G has a total edge irregular k-labeling and it is called *the total edge irregularity strength* of G, denoted by $tes(G)$. Ba$\check{c}$a, *et al*. have determined the total edge irregularity strength of path, cycle, stars, wheels, and friendship graph[2].

The original motivation for the definition of the total edge irregularity strength came from irregular assignments and the irregularity strength of a graphs introduced in[8]

An *irregular assignment* for any graph G is a t-labeling of the edges

$$\lambda : E \to \{1, 2, 3, \cdots, t\}$$

such that the weight of all vertices of G are distinct, where E is the edge set of G. The weight of vertex v in G under t-labeling λ is the sum of the label of v and the label of all edges incident to v. The smallest t for which there is an irregular assignment on G is called the *irregularity strength* of G.

Some results have been obtained for several classes of graphs. For instances, in[3,4,6] determined the exact value of the irregularity strength of trees. For special kinds of tree, in[7] have determined the irregularity strength of full $d - ary$ trees.

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We are interested in the total edge irregular labeling mainly because of the following intriguing theorem posted by[5] and conjecture posed by[10].

Theorem A. *Let $G=(V,E)$ be a graph with maximum degree Δ. Then*

$$tes(G) \geq \max\left\{\left\lceil \frac{|E(G)|+2}{3} \right\rceil, \left\lceil \frac{\Delta+2}{2} \right\rceil\right\}.$$

Conjecture B. *For every graph $G=(V,E)$, different from K_5, with the maximum degree Δ, then*

$$tes(G) = \max\left\{\left\lceil \frac{|E|+2}{3} \right\rceil, \left\lceil \frac{\Delta+1}{2} \right\rceil\right\}.$$

Many authors have been verified the conjecture. In[13,14] we have determined the total edge irregularity strength of the corona product of a path with certain graphs and nC_3-snake. Besides that, in[1,2] have determined the total edge irregularity strength of a categorical product of two paths and certain family of graphs. In[11] determined the total edge irregularity strength of complete graphs and complete bipartitegraphs. The total edge irregularity strength of the grids has determined by[12]. In[15] determined the total edge irregularity strength of subdivision of star. The total edge irregularity strength of the disjoint union of helm graphs have determined by[16].

As main result in this paper, we determined the total edge irregularity strength of corona product of cycle with a vertex.

2. Main Result

The corona product is one of many graph operations in graph theory. Formally, the corona product of a graph$G$ with a graph H, denoted by $G\odot H$, is the graph obtained by taking one copy G and $|V(H)|$ copies of H, and joining the $i-th$vertex of$G$with an edge to every vertex in $i-th$ copy of H[10]. Specially, in this paper we study about the corona product of m copies of cycle C_3 with a vertex, denoted by W_3^m.

First, we define the vertex set $V(W_3^m)$ and edge set $E(W_3^m)$ of W_3^m as follows:

$V(W_3^m)=\{x, x_j^i | i=1,2,\cdots,m; j=1,2,3\}$ and

$E(W_3^m)=\{x_j^i x_{j+1}^i, x_1^i x_3^i, x x_j^i, x x_3^i | i=1,2,\cdots,m; j=1,2\}$.

Theorem 1. *For $m\geq 2$, $tes(W_3^m)=2m+1$.*

Proof. By definition of the edge set of W_3^m, the number of edges of W_3^m is $6m+2$. By Theorem A, we find that $tes(W_3^m)\geq 2m+1$. To find that $tes(W_3^m)\leq 2m+1$, $\{x_0 y_{i,j} \mid i=1,\ldots,m; j=1,2,$ we construct a total edge irregular k-labeling for W_3^m where $k=2m+1$. Define a total labeling f for W_3^m as follows:

- 1. $f(x)=2m+1$,
- for $i=1,2,\cdots,m$,

$$f(x_j^i)=\begin{cases} j, & \text{for } j=1 \\ i, & \text{for } j=2 \\ m+i, & \text{for } j=3 \end{cases}$$

- for $i=1,2,\cdots,m$,

$$f(x_j^i x_{j+1}^i)=\begin{cases} j, & \text{for } j=1 \\ m+2-i, & \text{for } j=2 \end{cases}$$

- for $i=1,2,\cdots,m$,

$$f(x_1^i x_3^i)=1,$$

- for $i=1,2,\cdots,m$,

$$f(x x_j^i)=\begin{cases} m+i, & \text{for } j=1 \\ 2m+1, & \text{for } j=2,3 \end{cases}.$$

By the definition of the total labeling f, we find that $f: V\cup E\to\{1,2,\cdots,k\}$ where $k=2m+1$. The find that $f: V\cup E\to\{1,2,\cdots,k\}$ is the total edge irregular labeling, we have to show that the weight of all edges are distinct. By the definition of f, the weight of all edges of W_3^m as follows:

- for $i=1,2,\cdots,m$

$$wt(x_1^i x_2^i)=2+i,$$

- for $i=1,2,\cdots,m$

$$wt(x_1^i x_3^i)=m+2+i,$$

- for $i=1,2,\cdots,m$

$$wt(x_2^i x_3^i)=2m+i+2,$$

- for $i=1,2,\cdots,m$

$$wt(x x_1^i)=3m+2+i,$$

- for $i=1,2,\cdots,m$

$$wt(xx_2^i) = 4m + 2 + i,$$

- for $i = 1, 2, \cdots, m$

$$wt(xx_3^i) = 5m + 2 + i.$$

Accordingly, we find that every two edges different in W_3^m for $m \geq 2, e_1 \neq e_2$, we have $wt(e_1) \neq wt(e_2)$. This showed that, the total labeling $f : V \cup E \to \{1, 2, \cdots, k\}$, where $k = 2m + 1$, is a total edge irregular labeling for W_3^m. That is $tes(W_3^m) \leq 2m + 1$. □

3. Conclusion

In this paper we verified that the conjecture posed by Ivan $\check{c}$o and Jendrol' is true for the network constructed by some copies of cycle on three vertices corona a vertex.

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