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Simulation on Control Chart in Monitoring the Multivariate Process Variability

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Abstract. Control chart used to monitor shifts in the process mean and variability for multivariate data. This paper aims to monitor process variability through $\sqrt{|\boldsymbol{S}|}$ chart. Process variability observed by used the determinant of the sample covariance matrix, which is called $|\boldsymbol{S}|$. The result of this paper show that the improved $\sqrt{|\boldsymbol{S}|}$chart is more sensitive to shifts in the process variability. The standard $\sqrt{|\boldsymbol{S}|}$ chart has a bigger ARL than the improved chart by simulation.

INTRODUCTION

Control chart is a tool that used to monitoring process so the out-of-control signal can be detected. Montgomery (2009) describes the use of control chart use to monitor two thing in process, namely mean and variability. However, control charting procedure to monitor the process variability has received less attention in research. Many authors had focus on detect a shift in the process mean. But, it is important to monitor the variability, besides the mean in process. In 1988, Alt and Smith proposed a control chart for monitor the variability in process called generalized variance, denoted $|\boldsymbol{S}|$. Then, Djauhari (2005) continued the research to monitor the variability. Hamed (2014) and Rao et al (2013) showed the application of monitoring variability on hotmetal and fertilizers production, respectively.

Djauhari developed control limit for $\sqrt{|\boldsymbol{S}|}$ chart and $|\boldsymbol{S}|$ chart which is called the improved chart. The effectiveness of control chart can compare by Average Run Length (ARL) value. But, ARL comparison through $\sqrt{|\boldsymbol{S}|}$ chart is not done. So, this paper will discuss about ARL comparison for $\sqrt{|\boldsymbol{S}|}$ chart. In Section 2, we explain about the standar $\sqrt{|\boldsymbol{S}|}$ chart and the improved $\sqrt{|\boldsymbol{S}|}$ chart. In Section 3, we compare the ARL's value of $\sqrt{|\boldsymbol{S}|}$ chart, and final conclusion in section 4.

$\sqrt{|\boldsymbol{S}|}$ CONTROL CHART

Let $X_1, X_2, \dots, X_n$ be a random sample from a multivariate normal distribution, $N_p(\mu, \Omega)$ where Ω is positive definite matrix. $|S|$ have a same distribution with $\frac{|\Omega_0|}{(n-1)^p}\prod_{k=1}^{p}\chi^2_{n-k}$ where χ^2_{n-k} is disjoint for $k = 1,2,\dots,p$, so we can write :

$$|\boldsymbol{S}| = \frac{1}{(n-1)^p}|\boldsymbol{\Omega}_0|\prod_{k=1}^{p}\chi^2_{n-k}$$

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$$|\boldsymbol{S}| = \frac{|\boldsymbol{\Omega}_0|}{(2n-2)^2}(\chi^2_{2n-4})^2$$

Moment function for $|\boldsymbol{S}|$is formulated by χ^2 distribution defined by:

$$E(|\boldsymbol{S}|^r) = \left(\frac{2}{n-1}\right)^{pr} |\boldsymbol{\Omega}_0|^r \prod_{k=1}^{p} \frac{\Gamma\left(r + \frac{n-k}{2}\right)}{\Gamma\left(\frac{n-k}{2}\right)}, \tag{1}$$

Control chart for $\sqrt{|\boldsymbol{S}|}$ chart is calculated with expectation value and three sigma limit by:

$$E\left(\sqrt{|\boldsymbol{S}|}\right) = \sqrt{|\boldsymbol{\Omega}_0|}$$

$$\operatorname{var}\left(\sqrt{|\boldsymbol{S}|}\right) = |\boldsymbol{\Omega}_0| \sqrt{\frac{(a_2 - a_1^2)}{a_1}}$$

where

$$a_1 = \left(\frac{2}{n-1}\right)^{\frac{p}{2}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-p}{2}\right)}$$

$$a_2 = \left(\frac{1}{n-1}\right)^{p} \prod_{k=1}^{p}(n-k)$$

so that, the standard $\sqrt{|\boldsymbol{S}|}$ chart is defined by :

$$\mathrm{LCL} = \sqrt{|\boldsymbol{S}|}\left(1 - \frac{3\sqrt{a_2 - a_1^2}}{a_1}\right)$$

$$\mathrm{UCL} = \sqrt{|\boldsymbol{S}|}\left(1 + \frac{3\sqrt{a_2 - a_1^2}}{a_1}\right), \tag{2}$$

For the improved chart, assume there is m samples and $\mathbf{S}_\mathrm{i}$ as sample covariance matrix for $\mathrm{i} = 1,2,\dots,\mathrm{m}$, then there is $\mathbf{V}_\mathrm{i} = \sqrt{|\mathbf{S}_\mathrm{i}|}$ and $\bar{\mathbf{V}}_\mathrm{i}$ is average from m samples in $\sqrt{|\mathbf{S}_\mathrm{i}|}$. Let statistic $(\mathrm{n}-1)\mathbf{S}$ have independent and Wishart distribution,$\mathrm{W}_\mathrm{p}(\boldsymbol{\Omega}, (\mathrm{n}-1))$, so that statistic $\mathrm{m}(\mathrm{n}-1)\bar{\mathbf{S}}$ also have distribution Wishart, $\mathrm{W}_\mathrm{p}(\boldsymbol{\Omega}, \mathrm{m}(\mathrm{n}-1)$, and that means:

$$|\bar{\boldsymbol{S}}| \sim \left[\frac{|\boldsymbol{\Omega}|}{\{m(n-1\}^p}\right] \prod_{k=1}^{p} \chi^2_{n-k} \tag{3}$$

And then, Equation (1), can write:

$$E(|\boldsymbol{S}|^r) = \left[\frac{2}{m(n-1)}\right]^{pr} |\boldsymbol{\Omega}|^r \prod_{k=1}^{p} \frac{\Gamma\left(r + \frac{m(n-1)-k+1}{2}\right)}{\Gamma\left(\frac{m(n-1)-k+1}{2}\right)}, \tag{4}$$

So, control chart for the improved $\sqrt{|\boldsymbol{S}|}$ chart is defined by:

$$\mathrm{LCL} = \sqrt{|\mathbf{S}|}\left(\frac{a_1}{b_1} - 3\sqrt{\frac{a_2 - a_1^2}{b_2}}\right)$$

$$\mathrm{UCL} = \sqrt{|\mathbf{S}|}\left(\frac{a_1}{b_1} + 3\sqrt{\frac{a_2 - a_1^2}{b_2}}\right) \tag{5}$$

where

$$b_1 = \left(\frac{2}{m(n-1)}\right)^{\frac{p}{2}} \frac{\Gamma\left(\frac{m(n-1)+1}{2}\right)}{\Gamma\left(\frac{m(n-1)-p+1}{2}\right)}$$

$$b_2 = \left(\frac{1}{m(n-1)}\right)^{p} \prod_{k=1}^{p}(m(n-1)-k+1)$$

ARL COMPARISON

In previous section, we have control limit for the standard chart and the improved chart. Now, we would to compare the ARL of the standard chart in Equation (2) with the improved chart in Equation (5).

Let the Δ be the shift from in-control value, and $\widehat{\Delta}$ as the estimate of Δ, $\sqrt{|\mathbf{S}|}$ chart. We define $\widehat{\Delta} = k^{*}\widehat{\sigma}_{u}$ and $\widehat{\Delta} = k\widehat{\sigma}_{s}$ as the estimator for standard chart and the improved chart, respectively, where

$$\widehat{\sigma}_{u} = \sqrt{|\mathbf{S}|}\frac{\sqrt{a_2 - a_1^2}}{a_1}$$

$$\widehat{\sigma}_{s} = \sqrt{|\mathbf{S}|}\sqrt{\frac{a_2 - a_1^2}{b_2}}$$

If k^{*}constant, we can write $k^{*} = k\left(\frac{\widehat{\sigma}_{u}}{\widehat{\sigma}_{s}}\right)$, so we find the relationship between k and k^{*} are:

$$k^{*} = k \cdot \frac{\sqrt{b_2}}{a_1}.$$

We use k be the shift in ARL, which is use β value to observe the out-of-control. Let ARL(u) and ARL(s) represent the estimate of ARL for individual and subgroup observation, respectively, so we can compare the estimate of the ARL's value of the $\sqrt{|\mathbf{S}|}$ chart.

If$\Phi(x)$ is the cumulative distribution function of the standard normal distribution, then:

$$\text{ARL(u)} = \frac{1}{1 - \beta_u}$$

where $\beta_u = \Phi(3 - k) - \Phi(-3 - k)$,

$$\text{ARL(s)} = \frac{1}{1 - \beta_s}$$

where $\beta_u = \Phi(3 - k^{*}) - \Phi(-3 - k^{*})$.

Next, we compare the ARL value from the standard chart and the improved chart. Given k, $0 < k < 2$ in the step of 0.25 where $m = 20$, $n = 5, 10, 25, 50,$ and100, and then for p variable, we use $p = 2$ and 3. In Table 1, we can compare the ARL standard and the improved, when k is fixed, and Tabel 2, when k^{*} is fixed.

TABEL 1. ARL(s), when ARL(u) given.

k	ARL(u)	n = 5		n = 10		n = 25		n = 50		n = 100	
		p = 2	p = 3	p = 2	p = 3	p = 2	p = 3	p = 2	p = 3	p = 2	p = 3
		ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)	ARL(s)
0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
0.25	281.15	235.91	26.07	264.07	196.68	275.22	256.15	278.31	269.91	279.76	275.79
0.5	155.22	101.72	3.40	132.50	69.11	146.93	123.08	151.20	139.88	153.24	147.70
0.75	81.22	44.55	1.33	64.51	26.59	74.92	58.09	78.13	69.75	79.69	75.49
1	43.89	21.29	1.03	33.11	11.77	39.74	29.18	41.84	36.41	42.88	40.11
1.25	24.96	11.17	1.00	18.16	5.98	22.29	15.78	23.64	20.20	24.30	22.53
1.5	14.97	6.43	1.00	10.65	3.47	13.25	9.18	14.11	11.93	14.54	13.41
1.75	9.47	4.03	1.00	6.67	2.27	8.34	5.74	8.91	7.49	9.19	8.44
2	6.30	2.75	1.00	4.45	1.66	5.55	3.85	5.93	4.99	6.12	5.62

TABEL 2. ARL(u), when ARL(s) given.

k*	ARL(s)	n = 5		n = 10		n = 25		n = 50		n = 100	
		p = 2	p = 3	p = 2	p = 3	p = 2	p = 3	p = 2	p = 3	p = 2	p = 3
		ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)	ARL(u)
0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
0.25	281.15	314.31	365.74	296.17	331.44	286.81	301.96	283.93	291.46	282.53	286.30
0.5	155.22	211.48	352.38	178.42	249.14	163.58	188.32	159.27	170.78	157.21	162.80
0.75	81.22	130.74	331.96	100.14	171.69	87.81	108.83	84.37	93.69	82.76	87.18
1	43.89	80.24	306.69	57.02	115.27	48.36	63.36	46.02	52.44	44.93	47.93
1.25	24.96	50.24	278.86	33.69	77.59	27.87	38.08	26.34	30.59	25.63	27.59
1.5	14.97	32.36	250.39	20.76	52.97	16.87	23.76	15.87	18.67	15.40	16.69
1.75	9.47	21.48	222.75	13.35	36.84	10.73	15.41	10.06	11.93	9.75	10.60
2	6.30	14.70	196.84	8.95	26.13	7.15	10.38	6.70	7.98	6.50	7.07

From Table 1, we see clearly that, the ARL(u) is bigger than ARL(s). The result in Table 2 is also show the same result. When k* fixed, ARL for standard chart is bigger than ARL for improved chart.

For ARL value, we use β-error. If ARL value getting smaller, it means the effectiveness the chart getting bigger. So that, the improved chart more effective to use for monitor variability in process, if compared to the standard $\sqrt{|\boldsymbol{S}|}$ chart.

CONCLUSIONS

The result show that the ARL of $\sqrt{|\boldsymbol{S}|}$ chart is same from the the ARL of $|\boldsymbol{S}|$ chart. The improved $\sqrt{|\boldsymbol{S}|}$ chart is more effective than the standard $\sqrt{|\boldsymbol{S}|}$ chart. It is suggested, in the future research, we would investigate the effectiveness between $\sqrt{|\boldsymbol{S}|}$ chart and $|\boldsymbol{S}|$ chart.

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