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Stability analysis of two predators and one prey population model with harvesting in fisheries management

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Abstract. The discussion is focussed in the interaction between two predators and one prey population model in fishery management. Mathematically model is built by involving harvesting with constant efforts in the two predators and one prey populations. The positive equilibrium point of the model is analyzed via linearization and Routh-Hurwitz stability criteria. From the analysis, there exists a certain condition that makes the positive equilibrium point is asymptotically stable. The stable equilibrium point is then related to the maximum profit problem. With suitable value of harvesting efforts, the maximum profit is reached and the predator and prey populations remain stable. Finally, a numerical simulation is carried out to find out how much the maximum profit is obtained and to visualize how the trajectories of predator and prey tend to the stable equilibrium point.

1. Introduction

This study discusses the stability analysis of two predators and one prey population model in fishery populations with constant harvesting efforts. An example of such as fishing population interaction model might be found at Mahalona Lake, Matano Lake, and Towuti Lake. These three lakes previously have an endemic fish in their ecosystem, namely Buttini fish (glossogobius matanansis) [1] which are native fish at Mahalona Lake, Matano Lake, and Towuti Lake which are located in East Luwu Regency, South Sulawesi Province, which borders directly with Morowali Regency, Central Sulawesi Province [2-3]**.** The three lakes are well known and become an area that represents the tectonic lake ecosystem with beautiful natural panorama, crystal clear water flow and relatively calm of water waves.

Because of a high quality taste of this endemic fish, fishing communities around the lake often catch fish or harvest by damaging the environment. It is feared that there will be extinction in the future [4]. Some of the causes that make fish populations, including endemic fish are extinct, namely the intensity of fishing that causes over fishing and over exploitation and also the introduction of fish released by the community in this case Mujair fish (oreochromis mossambica) and Nila fish (oreochromis nilotichus) [5]. Nowadays, the introduced fish became the dominant fish in the lake habitat so that the endemic fish began to be threatened and it was feared that the fish would experience the extinction.

The interaction between predator prey populations in fish populations [6] where Butini fish (B) as prey, Mujair fish (M) and Nila fish (N) as predators formed in mathematical modeling that aims to understand the dynamics generated from fishing activities or harvesting fish and obtaining optimal income, taking into account ecological aspects in maintaining the ecosystem and sustainability of some of these species [6-7]. Mathematical modeling that is suitable for use in this study is the predator-prey model with predators namely Nila fish and Mujair fish, with prey namely Butini fish that follow the logistic growth model. This simplest model was introduced by A.J. Lotka in 1925 and V. Volterra in 1926 and also known as the Lotka-Volterra model which describes interactions between predator populations or competitive interactions between populations [8-9].

There have been many studies conducted by previous researchers on the use of predator prey models in several research problems based on the assumptions and parameters used, as well as research that discusses the application of dynamic reaction models on predator prey populations with structural stages in predators [10-11]. Then, with the addition of the assumption of adding time delay in the behavior of predators [12-13], conduct quantitative analysis by adding harvesting assumptions with Holling type II and Holling type III response functions [14-17], the optimal harvesting policy for predator prey populations [18-20], and the assumption of linear harvesting in the population of prey predators as a policy in economic aspects that can benefit the community [21- 23].

Most mathematical models that consider economic aspects in the field of fisheries, do not just focus on fishing or harvesting activities. However, it also takes into account other variables including natural conditions (weather), ships used, fishing gear that do not damage the environment, and other equipment needed [4]. After harvesting activities are carried out, it is also necessary to pay attention to other economic aspects, namely the equilibrium between supply and demand in setting prices for fisheries resources that affect the profits or profits obtained [7]. Mathematically, to obtain the maximum profit (π) based on total revenue (TR) and total cost (TC) of harvesting activities carried out in the three predator prey populations [24-25].

The predator prey model that is analyzed follows the logistic growth model based on the assumption of harvesting in the two predators and one prey populations model. The first step taken is to determine the equilibrium point then an equilibrium point stability analysis is carried out to determine the behavior of the system for a certain period of time [20]. Next, a bionomic equilibrium from harvesting activities will be determined in the three predator prey populations [25, 26], and model simulation using Maple software. The simulation is carried out with the assumption that all the three populations are of economic value and can increase income for the community, so that the three populations still exist, maintain sustainability, and remain stable, even though exploited or harvested by humans. Based on the description, the purpose of this study is to analyze the stability of the predator prey models in fishery populations and harvesting in all the three predator prey populations.

2. Predator-prey population model

The results of the study show that the predator prey model in the fishery population (*B, M, N*) by the logistical equation in the prey model $\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right)$, and harvesting occurs in the predator prey populations, with the parameter E_i where i = 1, 2, 3 which states the harvesting effort imposed on the predator prey population, with reference to the study [27] with the assumption of additional interactions between predator prey without involves the response function type Crowley-Martin function, as in Equation (1).

$$\begin{aligned}
\frac{dB}{dt} &= rB\left(1 - \frac{B}{K}\right) - \alpha MB - \beta NB - E_1 B \\
\frac{dM}{dt} &= -bM + \delta MB - dM - E_2 M \\
\frac{dN}{dt} &= -cN + \theta NB + dM - E_3 N.
\end{aligned} \tag{1}$$

The description of the parameters used are K is the carrying capacity; r is the intrinsic growth rate in the prey; α is the interaction of Mujair fish with Butini fish; β is the interaction of Nila fish with Butini fish; b denotes the natural death of Mujair fish; δ is the interaction of Mujair fish with Butini fish; *c* denotes the natural death of Nila fish; θ is the interaction of Butini fish with Nila fish; and *d* denotes the conversion rate *M* variable with positive parameter.

By letting $r_1 = r - E_1, r_2 = b + d + E_2$, and $r_3 = c + E_3$, Equation (1) is written as

$$\frac{dB}{dt} = r_1 B\left(1 - \frac{B}{K}\right) - \alpha MB - \beta NB$$
$$\frac{dM}{dt} = -r_2 M + \delta MB \tag{2}$$
$$\frac{dN}{dt} = -r_3 N + \theta NB + dM.$$

From the Equation (2), four non-negative equilibrium points are obtained. These equiblirium points

$$TE_1(B, M, N) = (0, 0, 0),$$
$$TE_2(B, M, N) = (K, 0, 0),$$
$$TE_3(B, M, N) = \left(\frac{r_3}{\theta}, 0, \frac{r_1(K\theta - r_3)}{K\beta\theta}\right), \text{ and}$$
$$TE_4(B, M, N) = \left(\frac{r_2}{\delta}, \frac{r_1(K\delta - r_2)(r_3\delta - r_2\theta)}{K(\alpha\delta r_3 - \alpha r_2\theta + \beta d\delta)\delta}, \frac{dr_1(K\delta - r_2)}{K(\alpha\delta r_3 - \alpha r_2\theta + \beta d\delta)}\right).$$

The equalibrium ponit can be written as

$$TE_4(B, M, N) = \left(\frac{r_2}{\delta}, \frac{r_1 S(r_3\delta - r_2\theta)}{KT\delta}, \frac{dr_1 S}{KT}\right),$$

where $S = K\delta - r_2$ and $T = \alpha\delta r_3 - \alpha r_2\theta + \beta d\delta$.

Equilibrium point $TE_4(B, M, N)$ is a positive equilibrium point when $S = K\delta - r_2 > 0$ and $T = \alpha\delta r_3 - \alpha r_2\theta + \beta d\delta > 0$. So, in conducting a stability analysis, a non-negative equilibrium point is used, namely point $TE_4(B, M, N)$. Equilibrium point $TE_4(B, M, N)$ is obtained by solving the system of equations $\frac{dB}{dt} = 0$, $\frac{dM}{dt} = 0$, and $\frac{dN}{dt} = 0$. Since we just consider the local stability of the equilibrium point, the model is linearized using the Jacobian matrix

$$J = \begin{pmatrix} r_1 - \frac{2r_1 B}{K} - \alpha M - \beta N & -\alpha B & -\beta B \\ M\delta & B\delta - r_2 & 0 \\ N\theta & d & B\theta - r_3 \end{pmatrix}.$$

From the Jacobian matrix, we substitute the equilibrium point and define the characteristic equation by the equation $\det(\mathbf{J} - \lambda\,\mathbf{I}) = 0$ which can be written as $f(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where the coefficients of the function are related to the entries of Jacobian matrix. Based on the Routh-Hurwitz stability criteria, the equalibrium point $TE_4(B, M, N)$ is asymptotically stable if and only if $a_0 > 0$, $a_2 > 0$, and $a_2 a_1 - a_0 > 0$ [19].

3. Bionomic equilibrium

Bionomic equilibrium aims to combine ecological equilibrium and economic equilibrium with the assumption of providing added value to human life [26]. Equilibrium point $TE_4(B, M, N)$ in Equation (2) which shows the harvesting effort in the three predator prey populations based on that assumption $S = K\delta - r_2 > 0$ and $T = \alpha\delta r_3 - \alpha r_2\theta + \beta d\delta > 0$, with $r_1 = r - E_1, r_2 = b + d + E_2$, and $r_3 = c + E_3$

then $TE_4(B, M, N) = \left(\frac{r_2}{\delta}, \frac{r_1 S(r_3\delta - r_2\theta)}{KT\delta}, \frac{dr_1 S}{KT}\right)$ can be written,

$$TE_4(B^*, M^*, N^*) = \left(\frac{r_2(r - E_1)}{\delta}, \frac{r_1 S(r_3\delta - r_2\theta)(b + d + E_2)}{KT\delta}, \frac{dr_1 S(c + E_3)}{KT}\right).$$

Next, determining the total revenue function (TR), total cost function (TC), and maximum profit (π) on harvesting efforts (E_1, E_2, and E_3) [24, 28]. Suppose the unit price for the population stock (*B, M, N*) is symbolized by p_1, p_2, and p_3. The total revenue function (TR) as shown,

$$TR\ (B, M, N) = TR\ (B) + TR\ (M) + TR\ (N) \tag{3}$$

$$= p_1E_1B + p_2E_2M + p_3E_3N \;.$$

Substituting the values of B^*, M^*, dan N^* in Equation (3) to obtain,

$$TR = p_1E_1\left(\frac{r_2(r-E_1)}{\delta}\right) + p_2E_2\left(\frac{r_1S(r_3\delta-r_2\theta)(b+d+E_2)}{KT\delta}\right) + p_3E_3\left(\frac{dr_1S(c+E_3)}{KT}\right)$$

$$= p_1E_1\left(\frac{r_2r-r_2E_1}{\delta}\right) + p_2E_2\left(\frac{(r_1Sr_3\delta-r_1Sr_2\theta)(b+d+E_2)}{KT\delta}\right) + p_3E_3\left(\frac{dr_1Sc+dr_1SE_3}{KT}\right)$$

$$= \frac{p_1r_2rE_1-p_1r_2E_1^2}{\delta} + p_2E_2\left(\frac{r_1Sr_3\delta b+r_1Sr_3\delta d+r_1Sr_3\delta E_2-r_1Sr_2\theta b-r_1Sr_2\theta d-r_1Sr_2\theta E_2}{KT\delta}\right) + \frac{p_3dr_1ScE_3+p_3dr_1SE_3^2}{KT}$$

$$= \frac{p_1r_2rE_1-p_1r_2E_1^2}{\delta} + \frac{p_2r_1Sr_3\delta bE_2+p_2r_1Sr_3\delta dE_2-p_2r_1Sr_2\theta bE_2-p_2r_1Sr_2\theta dE_2+p_2r_1Sr_3\delta E_2^2-p_2r_1Sr_2\theta E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3+p_3dr_1SE_3^2}{KT}$$

$$= \frac{p_1r_2rE_1-p_1r_2E_1^2}{\delta} + \frac{p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)E_2+p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3+p_3dr_1SE_3^2}{KT}$$

$$TR = \frac{p_1r_2rE_1}{\delta} - \frac{p_1r_2E_1^2}{\delta} + \frac{p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)E_2}{KT\delta} + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3}{KT} + \frac{p_3dr_1SE_3^2}{KT}. \quad (4)$$

The total cost function is assumed to be proportional to the catch from the harvesting effort (E_1, E_2, E_3), and the coefficient form of each catch (c_1, c_2, c_3). The total cost function (TC) can be written

$$TC = c_1E_1 + c_2E_2 + c_3E_3. \quad (5)$$

By substituting *TR* values in Equation (4) and *TC* values in Equation (5) into the equation of maximum profit (π) yields

$$\pi = TR - TC$$

$$\pi = \left(\frac{p_1r_2rE_1}{\delta} - \frac{p_1r_2E_1^2}{\delta} + \frac{p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)E_2}{KT\delta} + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3}{KT} + \frac{p_3dr_1SE_3^2}{KT}\right) - (c_1E_1 + c_2E_2 + c_3E_3)$$

$$= \frac{p_1r_2rE_1}{\delta} - (c_1E_1) - \frac{p_1r_2E_1^2}{\delta} + \frac{p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)E_2}{KT\delta} - (c_2E_2) + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3}{KT} - (c_3E_3) + \frac{p_3dr_1SE_3^2}{KT}$$

$$= \frac{p_1r_2rE_1-\delta c_1E_1}{\delta} - \frac{p_1r_2E_1^2}{\delta} + \frac{p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)E_2-KT\delta c_2E_2}{KT\delta} + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{p_3dr_1ScE_3-KTc_3E_3}{KT} + \frac{p_3dr_1SE_3^2}{KT}$$

$$\pi = \frac{(p_1r_2r-\delta c_1)E_1}{\delta} - \frac{p_1r_2E_1^2}{\delta} + \frac{(p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)-KT\delta c_2)E_2}{KT\delta} + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta} + \frac{(p_3dr_1Sc-KTc_3)E_3}{KT} + \frac{p_3dr_1SE_3^2}{KT}. \quad (6)$$

The maximum profit function is evaluated at the equalibrium point $TE_4(B^*, M^*, N^*)$ that depends on the harvesting effort, so it is necessary to know the critical point of the profit function by determining the first derivative of Equation (6) with respect to the efforts and obtained

$$\frac{\partial\pi}{\partial E_1} = \frac{(p_1r_2r-\delta c_1)E_1}{\delta} - \frac{p_1r_2E_1^2}{\delta},$$

$$\frac{\partial\pi}{\partial E_2} = \frac{(p_2r_1S(r_3\delta b+r_3\delta d-r_2\theta b-r_2\theta d)-KT\delta c_2)E_2}{KT\delta} + \frac{p_2r_1S(r_3\delta-r_2\theta)E_2^2}{KT\delta}, \text{ and} \quad (7)$$

$$\frac{\partial\pi}{\partial E_3} = \frac{(p_3dr_1Sc-KTc_3)E_3}{KT} + \frac{p_3dr_1SE_3^2}{KT}.$$

The critical point of Equation (6) is obtained by assuming the left side of Equation (7) with a value equal to zero, as explained below.

a. When $\frac{\partial\pi}{\partial E_1} = 0$, then $E_1 = \frac{p_1r_2r-\delta c_1}{2p_1r_2}$

b. When $\frac{\partial\pi}{\partial E_2} = 0$, then $E_2 = \frac{Sb\delta p_2 r_1 r_3 - Sbp_2 r_1 r_2\theta + Sd\delta p_2 r_1 r_3 - Sdp_2 r_1 r_2\theta - KT\delta c_2}{2p_1 r_1 S(r_3\delta - r_2\theta)}$

c. When $\frac{\partial\pi}{\partial E_3} = 0$, then $E_3 = \frac{p_3 d r_1 S c - KT c_3}{2p_3 d r_1 S}$.

The given critical value of profit function is expected to maximize the profit function and the equilibrium point $TE_4(B^*, M^*, N^*)$ remains asymptotically stable. In other words, when the predators and prey population are harvested at the critical efforts, the three populations will not be extinct and profit maximum is reached.

4. Numerical simulation

The parameter value used in this study is adapted from the parameter value which is taken from a number of references. In the numerical simulation, the value of parameters are given $K = 1000$; $r_1 = 1.5$; $r_2 = 1.4$; $r_3 = 1.5$; $\alpha = 0.4$; $\beta = 0.5$; $\delta = 0.5$; $\theta = 0.5$; $d = 0.3$ according to some previous studies [26]. These parameters are used to determine the equilibrium point of equation (2), from which we have the positive equilibrium point $T_1E_4(B^*, M^*, N^*) = (2.80000, 0.7872631579, 2.361789474)$. The associated Jacobian matrix at this equilibrium point is given by

$$J = \begin{pmatrix} -0.13540 & -1.12 & -1.4 \\ 0.39 & 0 & 0 \\ 1.315 & 0.3 & -0.10 \end{pmatrix}.$$

The characteristic equation $f(\lambda) = \lambda^3 + 0.2354\lambda^2 + 2.29134\lambda + 0.20748$ has eigen values $\lambda_1 = -0.7216396575 + 1.507643233i$; $\lambda_2 = -0.9107206849$; and $\lambda_3 = -0.7216396575 - 1.507643233i$. It is easy to see that the Routh-Hurwitz stability criteria are also satisfied. Therefore, the equilibrium point T_1E_4 is asymptotically stable [20].

In order to simulate the profit function, we take the parameter values $p_1 = 1.8$; $p_2 = 1.9$; $p_3 = 1.7$; $c_1 = 0.1$; $c_2 = 0.05$; $c_3 = 0.1$; $r = 1.6$; $b = 0.8$; $c = 1.3$ The positive equilibrium point of Equation (1) still depends on the parameter E_1, E_2, and E_3, then we have $T_2E_4(B^*M^*N^*)$, where,

$$B^* = 2.20 + 2E_2,$$

$$M^* = \frac{-0.0064(15625E_1E_2 - 15625E_1E_3 + 50E_2^2 - 50E_3E_2 - 3125E_1 - 24955E_2 + 24945E_3 + 4989)}{40E_2 - 40E_3 + 4989},$$

$$N^* = \frac{0.00960(3125E_1 + 10E_2 - 4989}{40E_2 - 40E_3 - 23}).$$

After substituting the equilibrium point, we have Jacobian matrix

$$J = \begin{pmatrix} 1.6 - 0.00320B - 0.4M - 0.5N - E_1 & -0.4B & -0.5B \\ 0.5M & 0.5B - E_2 - 1.1 & 0 \\ 0.5N & 0.3 & 0.5B - E_3 - 1.3 \end{pmatrix}.$$

The equilibrium point $T_2E_4(B^*M^*N^*)$ is then substituted into Equation (6) to obtain

$$\pi = 2\,(3.584\,B - 0.00358\,B^2 - 0.8966\,MB - 1.120\,NB - 2.24\,E_1B - 0.05)\,E_1 - 2.80\,(1.6\,B - 0.0016\,B^2 - 0.4\,MB - 0.5\,NB)\,E_1^2 + 0.021052\,(20.5672\,MB - 41.1345\,E_2M - 45.24795\,M - 2.375)\,E_2 - 0.78726\,(0.5\,MB - E_2M - 1.1\,M)\,E_2^2 + 0.0105\,(145.84\,NB - 291.68\,E_3N + 87.5043\,M - 379.1853\,N - 9.5)\,E_3 - 2.3617\,(0.5\,NB - E_3N + 0.3\,M - 1.3\,N)\,E_3^2.$$

A bionomic equilibrium occurs when the total revenues minus total costs equals zero. The profit function will be maximized at the positive equilibrium. After substituting the positive equilibrium point $TE_4(B^*, M^*, N^*)$, the profit function still depends on the harvesting efforts. The critical points for the profit function are obtained by solving the system of equations of first partial derivatives which respect to the efforts. Therefore, we have critical point $E_1 = 0.8099206350$, $E_2 = 0.5332865355$, and $E_3 = 0.6624531695$. It is easy to check that the critical point maximizes the profit function. Substituting the critical point into the equilibrium point to get

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$T_3E_4(B^*M^*N^*) = (3.266573072, 0.9172096395, 0.8359379835)$. The Jacobian matrix evaluated at the positive equilibrium point is given by

$$J = \begin{pmatrix} 0.00083936 & -1.28 & -1.6 \\ 0.455 & -0.03328653 & 0 \\ 0.415 & 0.3 & -0.36245317 \end{pmatrix}.$$

The characteristic equation associated with the Jacobian matrix can be written as $f(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$ where $a_0 = 0.4515848593$, $a_1 = 1.258132640$, and $a_2 = 0.3949003341$. We know that $a_2\, a_1 - a_0 > 0$ and the Routh-Hurwitz stability criteria are satisfied. Therefore, the equilibrium point $T_3E_4(B^*, M^*, N^*)$ is asymptotically stable.

From the analyses above, it follows that when the predator and prey population are harvested at the level of critical point of harvesting efforts, the populations remain stable which means that the populations will not be extinct for a long period of time. Besides that, the given efforts also maximize the profit function with value $\pi_{max} = 5.19899655$. When this analysis is used by the government in terms of managing the renewable natural resources, the endemic fish population, especially for Butini fish population will not become extinct and will provide maximum and sustainable benefits.

The following Figure (1), (2), and (3) show the dynamics of trajectories (solution curves) for the predators and prey populations against time (t). The initial value for populations are given by $B(0) = 4.0$, $M(0) = 1.5$, and $M(0) = 1.5$.

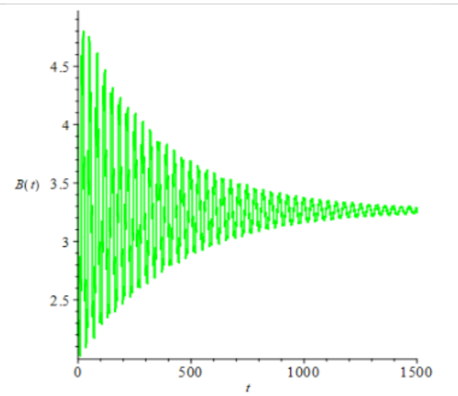

Figure 1. The behavior of solution curve of prey population (B)

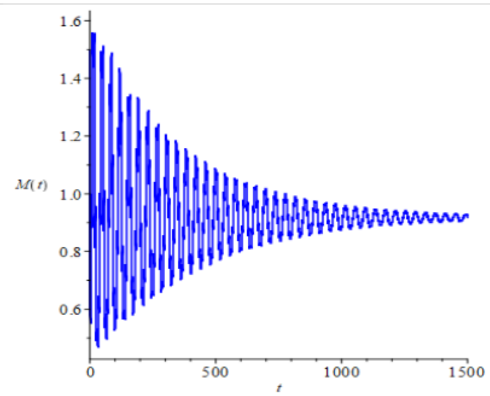

Figure 2. The behavior of solution curve of predator population (M)

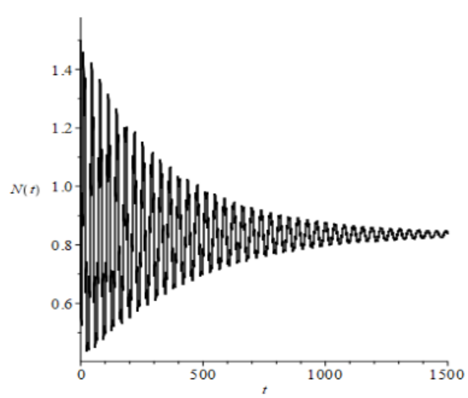

Figure 3. The behavior of solution curve of predator population (N)

5. Conclusions

The growth rate of two predator (*M*, *N*) and one prey (*B*) population model with constant harvesting effort for the three populations are investigated. The model actually has four non negative equilibrium points. Since we just focus on analyzing the sustainability of the population, the positive equilibrium point is only investigated and analyzed the existence and the stability. Without harvesting, the positive equilibrium point is asymptotically stable under a certain condition. When the populations are harvested with suitable value of harvesting efforts, the positive equilibrium exists and remains asymptotically stable.

The positive equilibrium point is then related to the maximum profit problem and the critical value of profit function is found. The critical value of harvesting efforts maximize the profit function. Besides that, the positive equilibrium point is also asymptotically stable. The two predators and one prey actually can be harvested with constant of harvesting efforts and will not cause the extinction for all populations. In the other words, the Butini fish, the Nila fish, and the Mujair fish as the prey and predators population in the ecosystem can be managed well and will give maximum profit for the human for a long period of time.

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