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Optimal harvesting and stability of predator prey model with Monod-Haldane predation response function and stage structure for predator

R A Pratama [1,a], **S Toaha** [1,b,*] **and Kasbawati** [1,c]

[1] Department of Mathematics, Hasanuddin University,
Jln. Perintis Kemerdekaan, KM 10, 90245, Makassar, Indonesia

[a] rianadepratama@gmail.com, [b] syamsuddint@gmail.com,
[c] kasbawati@gmail.com, [*] corresponding author

Abstract. This article deals with growth rate predator prey population model with stage structure for predator. The predator population is divided into two stages, mature and immature predator. Predation functional response in the model follows the Monod-Haldane type. Under consideration that the population is a valuable stock, the mature predator and prey population are then harvested with constant effort. This model is then analysed by found the equilibrium point and stabilities. There exists four equilibrium points and stability analysed is focused only for interior point. The stable interior point is related to the maximum profit problem. From the analyses we found that there exists a condition for the effort of harvesting so that the interior point is still stable and also obtained maximum profit.

Keywords: Predator prey model, Monod-Haldane function, stage structure, optimal harvesting

1. Introduction

In study of population dynamics, the growth rate of predator prey is one model that widely studied by some researchers. When there are two or more populations living in the same environment, there will be interaction between the populations and the interaction that occur most often is predation, that is, one population as a predator and the other as a prey. In the case of frequencies of interaction between prey and predator very often, the prey population will be extinct. One strategy to protect the prey from extinction due to predation is by controlling the growth rate of prey and its predator, [1].

The dynamics of predator prey models have been studied and developed by many researchers. There are researchers who consider more than two populations in the model, some researchers consider economic aspects and other aspects in the model. Prey as well as predator can be considered into two stages, immature and mature populations. Regarding to the stage structured for predator prey model. Research on predator prey models with stage structure was also studied in [2] with stage structure in prey population. Study the predator pretty model with stage structure in [3] was found that the coexistence of both populations are strongly influenced by the population on the stage structure. Study of

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population dynamics of predator prey with stage structure and some extensions can be found, for example in [4, 5, 6].

In ecological modeling, the population as a useful stock is then exploited. The predator prey model with harvesting also become a special topic. The predator prey model with harvesting has been studied in [7] and model with stage structure and harvesting has been studied in [8]. In this article, we develop a model proposed in [3] by incorporating selective harvesting factor to the prey and mature predator population. We analyze the existence of the interior equilibrium point and its stability. The stable equilibrium point is then linked to how maximizing the profit from exploitation of the populations.

2. Predator prey model with harvesting and stage structure

A predator prey model with stage structure for predator in [3] has been studied. The model follows predation function with Monod-Haldane type. In this model, mature predator population is assumed has interaction with prey population. Whereas for immature predator has not interaction directly with the prey. The immature predator has not reproductive abilities and it depends on the interaction of mature predator and prey population. The model is

$$\begin{aligned}\frac{dx}{dt} &= \rho_1 x\left(1-\frac{x}{k_1}\right)-\frac{\beta\, x\, z}{\phi+\tau\, x^2}\\ \frac{dy}{dt} &= \frac{c\,\beta\, x\, z}{\phi+\tau\, x^2}-(\alpha+\delta_1)\, y\\ \frac{dz}{dt} &= \alpha\, y-\delta_2\, z.\end{aligned} \tag{1}$$

The symbols $\frac{dx}{dt}, \frac{dy}{dt}$, and $\frac{dz}{dt}$ define the growth rate of prey, immature predator, and mature predator respectively. Constant ρ_1 is the intrinsic rate of prey population and k_1 is carrying capacity for the prey when there is no predator population. A simplification functional response of Monod Haldane is $\frac{\beta\, x}{\phi+\tau\, x^2}$, [9]. The symbols β and c state the rate of predation and efficiency of predation for growth rate of immature predator. The symbol α is the rate from immature predator becomes mature predator. The symbols δ_1 and δ_2 denote the rate mortality for immature and mature predator respectively.

Under consideration that the populations in the model are beneficial, then harvesting factor is incorporated in the growth rate of the population. We improve model (1) by considering selective harvesting in prey and mature predator populations. The model becomes

$$\begin{aligned}\frac{dx}{dt} &= \rho_1 x\left(1-\frac{x}{k_1}\right)-\frac{\beta\, x\, z}{\phi+\tau\, x^2}-q_1\, E_1\, x\\ \frac{dy}{dt} &= \frac{c\,\beta\, x\, z}{\phi+\tau\, x^2}-(\alpha+\delta_1)\, y\\ \frac{dz}{dt} &= \alpha\, y-\delta_2\, z-q_2\, E_2\, z.\end{aligned} \tag{2}$$

The symbols q_1 and q_2 denote catchability coefficient for prey and mature predator population respectively. The symbols E_1 and E_2 denote constant efforts of harvesting. There are four suitable equilibrium points to be analysed, namely adalah $T_0 = (0,0,0)$, $T_1 = (K,0,0)$, $T_2 = (x_*,\ y_*,\ z_*)$, and $T_3 = (x^*,\ y^*,\ z^*)$,

where $K = \frac{k_1(\rho_1 - q_1 E_1)}{\rho_1}$,

$$x_* = \frac{(\alpha c\beta)+\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau},$$

$$y_* = \frac{c\,(\rho_1-q_1 E_1)}{(\alpha+\delta_1)}\left(\frac{(\alpha c\beta)+\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)$$

$$-\frac{\rho_1\, c\, \alpha}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))\, k_1}\left(\frac{(\alpha c\beta)+\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)^2,$$

$$z_* = \frac{c\,\alpha(\,\rho_1 - q_1E_1)}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))}\left(\frac{(\alpha c\beta)+\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)$$

$$-\frac{\rho_1\, c\, \alpha}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))\, k_1}\left(\frac{(\alpha c\beta)+\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)^2.$$

$$x^* = \frac{(\alpha c\beta)-\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau},$$

$$y^* = \frac{c\,(\,\rho_1 - q_1E_1)}{(\alpha+\delta_1)}\left(\frac{(\alpha c\beta)-\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)$$

$$-\frac{\rho_1\, c\, \alpha}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))\, k_1}\left(\frac{(\alpha c\beta)-\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)^2,$$

$$z^* = \frac{c\,\alpha(\,\rho_1 - q_1E_1)}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))}\left(\frac{(\alpha c\beta)-\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)$$

$$-\frac{\rho_1\, c\, \alpha}{((\alpha+\delta_1)(q_2\, E_2+\delta_2))\, k_1}\left(\frac{(\alpha c\beta)-\sqrt{A}}{2\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau}\right)^2,$$

and

$$A = (\alpha c\beta)^2 - 4\,(\alpha+\delta_1)(q_2\, E_2+\delta_2)\tau\,\phi\,(\alpha+\delta_1)(q_2\, E_2+\delta_2).$$

There is only one possibe positive equilibrium point for the model (2), as well as for model (1). Here, only the positive equilibrium point will be analysed. The possible positive equilibrium point and also stable for model (2) is written as $T_3 = (x^*,\ y^*,\ z^*)$. The Jacobian matrix for model evaluated at equilibrium point T_3 is given by

$$J = \begin{bmatrix} J_{11} & 0 & J_{13} \\ J_{21} & J_{22} & J_{23} \\ 0 & J_{32} & J_{33} \end{bmatrix},$$

where $J_{11} = \rho_1 - 2\frac{\rho_1 x^*}{k_1} - \frac{\beta\, z^*}{\phi+\tau\, x^{*2}} + \frac{2\,\beta\, z^*\,\tau\, x^{*2}}{(\phi+\tau\, x^{*2})^2} - q_1E_1$, $J_{13} = -\frac{\beta\, x^*}{\phi+\tau\, x^{*2}}$, $J_{21} = \frac{c\,\beta\, z^*}{\phi+\tau\, x^{*2}} - \frac{2\,c\,\beta\, z^*\,\tau\, x^{*2}}{(\phi+\tau\, x^{*2})^2}$, $J_{22} = -\alpha-\delta_1$, $J_{23} = \frac{c\,\beta\, x^*}{\phi+\tau\, x^{*2}}$, $J_{32} = \alpha$, $J_{33} = -q_2E_2 - \delta_2$

The characteristic equation associated with the Jacobian matrix J is given by

$$f(\lambda) = \lambda^3 + A_1\lambda^2 + A_2\lambda + A_3,$$

where $A_1 = -(J_{11}+J_{22}+J_{33})$, $A_2 = J_{11}J_{22} + J_{11}J_{33} + J_{22}J_{33} - J_{23}J_{32}$ and $A_3 = J_{11}J_{23}J_{32} - J_{11}J_{22}J_{33} - J_{13}J_{32}J_{21}$. According to Hurwitz stability test [10], the equilibrium point T_3 is asymptotically stable when the conditions $A_1 > 0,\ A_2 > 0,\ A_3 > 0$, and $A_1A_2 > A_3$ are satisfied.

3. Maximum profit in population harvesting

The stable equilibrium point of the model is then related to maximizing the profit for exploitation of the populations. We suppose that the prey and mature predator population are harvested with constant effort. From the exploitation we consider revenue and cost function. We define total revenue as $TR = p_1\, x^*\, E_1 + p_2\, z^* E_2$ and total cost $TC = c_1E_1 + c_2E_2$. The Constants, while the constants c_1 and c_2 denote unit cost of harvesting for prey and mature predator respectively.

The equilibrium point T_3 becomes a positive equilibrium point when $(E_1, E_2)\ D$, where $D = \{(E_1, E_2)| 0 \le E_1 \le E_{1max}, 0 \le E_2 \le E_{2max}\}$ for some value of E_{1max} and E_{2max}. The profit function for harvesting of prey and mature predator at equilibrium point from the exploitation at the equilibrium point T_3 is given by

$$\pi(E_1, E_2) = p_1 x^* E_1 + p_2 z^* E_2 - c_1 E_1 - c_2 E_2 . \quad (3)$$

We need to maximize the the profit function (3) in the feasible region of D for some values of E_{1max} and E_{2max}. The critical values of the efforts E_1 and E_2 are determined by considering the first partial derivatives and the critical values of the efforts at domain D. The profit function the equilibrium point T_3 is

$$\pi(E_1, E_2) = p_1 x^* E_1 + p_2 z^* E_2 - c_1 E_1 - c_2 E_2$$

$$= \frac{1}{2} \frac{p_1 E_1 \left(\alpha c \beta - \sqrt{w E_2^2 + v E_2 + u}\right)}{t E_2 + s}$$

$$+ \frac{p_2 E_2}{4 (e_1 E_2^3 + e_2 E_2^2 + e_3 E_2 + e_4)} \Big(a_1 E_1 E_2 + a_3 E_2 - a_4 E_2 + a_2 - 2 a_5 E_1 E_2 \sqrt{w E_2^2 + v E_2 + u} -$$
$$a_6 - 2 a_7 \sqrt{w E_2^2 + v E_2 + u} - 2 a_8 E_1 \sqrt{w E_2^2 + v E_2 + u} + 2 a_9 E_2 \sqrt{w E_2^2 + v E_2 + u} +$$
$$2 a_{10} \sqrt{w E_2^2 + v E_2 + u} + a_{11} E_2^2 + a_{12} E_2 + a_{13}\Big) - c_1 E_1 - c_2 E_2 ,$$

where

$e_1 = \alpha\ t^2 k_1 + t^2 \delta_1 k_1$, $e_2 = 2 \alpha s t k_1 + \alpha t^2 \delta_2 k_1 + 2 s t \delta_1 k_1 + t^2 \delta_1 \delta_2 k_1$, $e_3 = \alpha\ s^2 k_1 + 2 \alpha s t \delta_2 k_1 + s^2 \delta_1 k_1 + 2 s t \delta_1 \delta_2 k_1$, $e_4 = \alpha s^2 \delta_2 k_1 + s^2 \delta_1 \delta_2 k_1$, $a_1 = 2 \alpha^2 c^2 \beta\ t k_1$, $a_2 = \alpha^3 c^3 \beta^2 \rho_1$, $a_3 = 2 \alpha^2 c^2 \beta s k_1$, $a_4 = 2 \alpha^2 c^2 \beta t k_1 \rho_1$, $a_5 = \alpha c t k_1$, $a_6 = 2 \alpha^2 c^2 \beta s k_1 \rho_1$, $a_7 = \alpha^2 c^2 \beta \rho_1$, $a_8 = \alpha c s k_1$, $a_9 = \alpha c t k_1 \rho_1$, $a_{10} = \alpha c s k_1$, $a_{11} = w \alpha c \rho_1$, $a_{12} = v \alpha c \rho_1$, $a_{13} = u \alpha c \rho_1$, $s = \alpha \tau \delta_2 + \tau \delta_1 \delta_2$, $t = \alpha \tau + \tau \delta_1$, $u = \alpha^2 c^2 \beta^2 - 4 \alpha^2 \phi \tau {\delta_2}^2 - 8 \alpha \phi \tau \delta_1 {\delta_2}^2 - 4 \phi \tau {\delta_1}^2 {\delta_2}^2$, $v = -8 \alpha^2 \phi\ \tau \delta_2 - 16 \alpha \phi \tau \delta_1 \delta_2 - 8 \phi \tau {\delta_1}^2 \delta_2$, and $w = -4 \alpha^2 \phi\ \tau - 8 \alpha \phi\ \tau \delta_1 - 4 \phi \tau {\delta_1}^2$.

4. Numerical simulations

For simulation on model (1) without harvesting, we take the parameters $\rho_1 = 1.5$, $k_1 = 100$, $\phi = 8$, $\beta = 0.16$, $c = 0.4$, $\alpha = 0.02$, $\tau = 0.001$, $\delta_1 = 0.01$, $\delta_2 = 0.2$, $E_1 = 0$ dan $E_2 = 0$ in appropriate units. We have positive equilibrium point $T_3 = (48.5480, 499.5783, 49.9578)$. The characteristic equation associated with Jacobian matrix evaluated at the equilibrium point is $f(\lambda) = \lambda^3 + 0.60696 \lambda^2 + 0.86699 \lambda + 0.25231$. From the characteristic equation, it is easy to check that the Routh-Hurwitz criterion follows and the eigen values are $\lambda_1 = -0.41089$, $\lambda_2 = -0.39129$, and $\lambda_3 = -0.15692$. All of eigen values are real and negative, then the equilibrium point T_3 is locally asymptotically stable. This means the prey and mature predator will sustain for a long period of time.

For simulation on model (2), we use the values of parameter $\rho_1 = 1.5$, $k_1 = 100$, $\phi = 8$, $\beta = 0.16$, $c = 0.4$, $\alpha = 0.02$, $\tau = 0.001$, $\delta_1 = 0.01$, $\delta_2 = 0.2$, $q_1 = 1$, $q_2 = 1$ in appropriate units. We get an interior equilibrium point $T_3 = (x^*, y^*, z^*)$, where

$$x^* = \frac{1}{2} \frac{0.00128 - \sqrt{A_1}}{0.000029 E_2 + 0.0000058},$$

$$y^* = \frac{6.896551725 (-E_1 + 1.5)(0.00128 - \sqrt{A_1})}{0.000029 E_2 + 0.0000058}$$

$$-\frac{0.001034482758\left(0.00128-\sqrt{A_1}\right)^2}{(E_2+0.2)(0.000029\,E_2+0.0000058)},$$

$$z^*=\frac{0.1379310345\,(-E_1+1.5)\left(0.00128-\sqrt{A}\right)}{(E_2+0.2)\,(0.000029\,E_2+0.0000058)}-\frac{0.001034482758\left(0.00128-\sqrt{A_1}\right)^2}{(E_2+0.2)(0.000029\,E_2+0.0000058)},$$

where $A_1=-0.000026912\,E_2^2-0.0000107648\,E_2+0.00000056192$.
From equation (3) we get

$$\pi(E_1,E_2)=\frac{50\,E_1\left(0.00128-\sqrt{A_2}\right)}{0.00003\,E_2\;+\;0.000006}-\frac{25\,E_2}{0.000000003\,E_2^3+0.0000000016\,E_2^2+0.00000000032\,E_2+0.000000000022}\big(0.00000614\,E_1\,E_2+\;0.0000012288\,E_1-0.00000023\,E_2+0.00000000706-\;0.000048\,E_1\,E_2\sqrt{A_2}-0.00002112\sqrt{A_2}-0.0000096\,E_1\sqrt{A_2}+0.000072\,E_2\sqrt{A_2}+0.0000003456\,E_2^2\big)-50\,(E_1+E_2)\,,$$

where $A_2=-0.0000288\,E_2^2-0.00001152\,E_2+0.0000004864$.

We consider the range of efforts as $D=\{(E_1,E_2)|0\le E_1\le 0.038, 0\le E_2\le 0.038\}$. The critical point of the efforts E_1 and E_2 are determined by using the first partial derivatives and the boundary of D. In domain D we get four critical points for $\pi(E_1,E_2)$, namely $P_1=(E_1,E_2)=(0.006485,0.00372)$ as stationary point (saddle point), $P_2=(E_1,E_2)=(0,0.02957)$, $P_3=(E_1,E_2)=(0.03800,0)$, $P_4=(E_1,E_2)=(0.038,0.03522)$, $P_5=(E_1,E_2)=(0.03800,0.03800)$. Profit function which is evaluated at the critical point are given as follow;

1) $(E_1,E_2)=(0.00649,0.00372)$ with profit $\pi=31.06066$.
2) $(E_1,E_2)=(0,0.02956)$ with profit $\pi=148.87029$.
3) $(E_1,E_2)=(0.03800,0)$ with profit $\pi=182.58245$.
4) $(E_1,E_2)=(0.03800,0.03522)$ with profit $\pi=408.16992$, and
5) $(E_1,E_2)=(0.03800,0.03800)$ with profit $\pi=398.33448$.

From these critical points we get profit maximum with profit $\pi=408.16992$. The equilibrium point associated with the critical point is $T_3=(75.6780,\ 329.7842,\ 28.0403)$. The characteristic equation from Jacobian matrix evaluated at this equilibrium point is $f(\lambda)=\lambda^3+1.12768\lambda^2+0.22874\,\lambda+0.000382$ which gives eigenvalues as $\lambda_1=-0.86319$, $\lambda_2=-0.26279$, and $\lambda_3=-0.00168$. This means that the equilibrium point T_3 is locally asymptotically stable. In this situation, the prey and mature predator populations will sustain for a long period of time.

For initial population around the equilibrium point T_3, we take $z(0)=45, y(0)=90$, and $z(0)=20$ we get trajectories for prey, immature, and mature predator with and without harvesting are given the following figures.

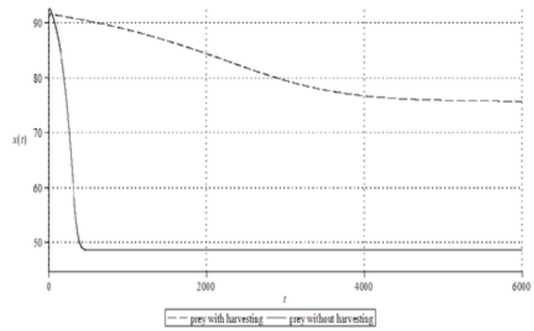

Figure 1. Trajectories for prey with and without harvesting

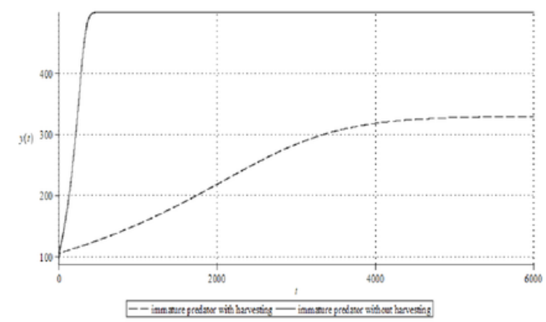

Figure 2. Trajectories for immature predator with and without harvesting

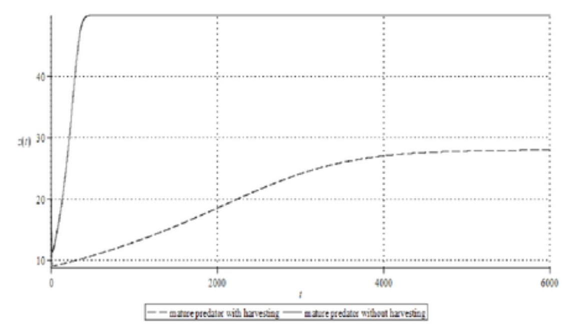

Figure 3. Trajectories for mature predator with and without harvesting

From Figure (1), (2), and (3) we know that all populations will sustain for a long time. However, there is a change in the value of the equilibrium point without harvesting $T_3 = (48.5480, 499.5783,\ 49.9578)$ and equilibrium point without harvesting $T_3 = (75.6780,$ $329.7842,\ 28.0403)$. The value of equilibrium point for prey and mature predator are decreasing but for immature predator is increasing in value.

5. Conclusions

One model of predator and prey populations with stage structure for predator population and Monod-Haldane functional response has been developed by adding harvesting factor to the prey and the mature predator populations. We found four equilibrium points, but only one is possible to become an interior equilibrium point. With suitable value of parameters, the interior equilibrium point without harvesting becomes locally asymptotically stable.

For the model with constant effort of harvesting, the stable equilibrium point is then linked to the problem of maximizing the profit function. We found a condition for the value of harvesting efforts that provides maximum profit and the interior equilibrium point remains stable. This means that the value of harvesting efforts keeps the prey and predator population sustainable and provides maximum profit for a long period of time.

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