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Abstract. This article deals with the dynamics of prey and predator populations in a two patch environment, a protected area and an unprotected area for fishing. The prey disperses between the two patches and migrates easily. There are two predators, one is in the protected area and another is in the unprotected area. The predators cannot migrate. Both prey and predator in unprotected area are harvested with constant efforts. The dynamical behavior of the populations is stated as a system of differential equations. The existence of a positive equilibrium point and its stability are investigated. We discuss the local stability of the positive equilibrium point. The stable equilibrium point is then associated with optimal harvesting problems. Based on the analysis, we found that there exist a stable positive equilibrium point when there is no harvesting. For model with constant efforts for both prey and predator, we found that over fishing will maximize the profit but the predator in the unprotected area will be extinct. With the help of Pontryagin's maximum principle in maximizing the present value of revenues, we found the extremal of the efforts that maximize the present value of revenues. This means that both prey and predator in the protected area as well as the prey and predator in the unprotected area are possibly coexist although the prey and the predator in the unprotected area are harvested with constant efforts. Some numerical simulations area given to confirm the result of analysis.

Keywords: Predator-prey, two patch environment, stability, protected area, present value of revenue

1. Introduction

The study of dynamical behavior of prey and predator populations does not only focus on how to control the population from the extinction. Fishery industries need some strategies to manage the population in order to get the optimal harvesting and the population will not lead to the extinction. In the exploitation activities, there are many things that need to be controlled so that the populations as valuable stocks can be well managed.

In the study of prey and predator models, some authors considered many factors in the models in the various perspectives. Some models with harvesting, migration, diffusion, stage structure, or harvesting and tax, have been studied in details by many authors. Some researchers considered the models with one prey and one predator, two preys and one predator, or prey with stage structure and one predator, with and without harvesting.

The prey-predator model with stage structure for predator in a two patch environment with harvesting in the unprotected area was discussed in [1] and determined a certain conditions to get an optimal harvesting. A model with harvesting in reserve area was considered and inserted tax as a control to prevent over explotation of the population [2] and harvesting to get the maximum value of present value of revenues [3,4].

Becuase of the complexity of dynamics of prey and predator fisheries, some researchers considered the selective harvesting in the models. Those who have considered selective harvesting of the prey, for examples in [2, 5, 6]. The studies of prey and predator models with selective harvesting of the predator, for examples in [7, 8, 9, 10]. The studies of prey and predator models with both prey and predator are harvested, for examples in [4, 11]. The dynamics of prey and predator fishery models in a two patch environment, protected area and unprotected area for any kind of fishing, with harvesting in the unprotected area and problems of maximizing the profit as well as present value of revenues have been discussed in detail by many authors, for examples in [1, 2, 3, 12, 13, 14, 15].

In this article, we consider the dynamical behavior of prey and predator populations in a two patch environment, namely a protected area and an unprotected area for fishing. The prey disperses and migrates in the two areas. There are two kinds of predators, one is in the protected area and the other is in the unprotected area. We suppose the predators can not migrate. The model includes four differential equations. Both prey and predator populations in the unprotected area are then harvested with constant efforts. We analyse the existence and the stability of the positive equilibrium point. The stable equilibrium point is then associated with the problem of maximizing profit as well as present value of revenues. The critical points of the efforts are determined in order to get the maximum profit as well as maximum present value of revenues. The Routh-Hurwitz criteria are referred to determine the stability of the equilibrium point. With the help of the Pontryagin's maximum principle, the optimal harvesting policy of present value of revenues can be done.

2. The dynamics of prey and predator in a two patch environment

We consider a prey-predator fishery management in a two patch environment, namely an unprotected area for fishing and another is a protected area for any kind of fishing. The prey population disperses in the two areas and can migrate easily from one area to another area. There is a predator preys the prey in the unprotected area and there exists also another predator preys the prey in the protected area. The two patches of environment are supposed to have the same characteristics. The growth rate of the prey is assumed to be logistic. The dynamical behavior of prey and predator populations is constituted as a system of four differential equations

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\tau_1 x+\tau_2 y-\alpha_1 xz \\
\frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\tau_1 x-\tau_2 y-\alpha_2 yw \\
\frac{dz}{dt} &= \beta_1\alpha_1 xz-k_1 z \\
\frac{dw}{dt} &= \beta_2\alpha_2 yw-k_2 w.
\end{aligned} \tag{1}$$

The symbols $x = x(t)$ and $y = y(t)$ constitute the size of preys population in the unprotected area and in the protected area at time t, respectively. The symbols $z = z(t)$ and $w = w(t)$ constitute the size of predators population in the unprotected area and in the protected area at time t. Parameters r and s state the intrinsic growth rate for the preys in the unprotected area and in the protected area. Parameters K and L state the carrying capacity of the environment for the preys in the unprotected area and in the protected area. Parameters α_1 and α_1 measure the intensity interaction between the prey and predator in the unprotected area and in the protected area. The values of β_1 and β_2 which take the value from zero to one measure the effect of predation to the predator in the unprotected area and in the protected area. Parameter τ_1 defines the migration rate from the prey in the unprotected area to the prey in the protected area and parameter τ_2 defines otherwise. Parameters k_1 and k_2 denote the mortality rate for the predators in the unprotected area and in the protected area. All parameters of the model are supposed to be positive. Since the model constitutes the dynamics of the prey and predator populations then we just consider the value of populations as $x(t)\geq 0$, $y(t)\geq 0$, $z(t)\geq 0$, and $w(t)\geq 0$.

Under consideration that the prey and predator populations are valuable stocks, the prey and predator in the unprotected area are exploited with constant efforts. The dynamical behavior of the prey and predator populations is then extended and written in the form of

$$\begin{aligned}
\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\tau_1 x+\tau_2 y-\alpha_1 xz-q_1E_1x \\
\frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+\tau_1 x-\tau_2 y-\alpha_2 yw \\
\frac{dz}{dt} &= \beta_1\alpha_1 xz-k_1z-q_2E_2z \\
\frac{dw}{dt} &= \beta_2\alpha_2 yw-k_2w.
\end{aligned} \tag{2}$$

In the model (2), parameters q_1 and q_2 state the cathability coefficient for the prey and predator populations respectively. Parameters E_1 and E_2 state the constant efforts of harvesting which satisfy $0\leq E_i\leq E_{i\max}$ for $i=1, 2$ and a certain value of $E_{i\max}$. There are five equilibrium points suitable to be analysed, namely $(0, 0, 0, 0)$, $(x_+, y_+, 0, 0)$, $(x_+, y_+, z_+, 0)$, (x_+, y_+, z_+, w_+), and $(x_+, y_+, 0, w_+)$. There is only one possibe positive equilibrium point for the model (2), as well as for model (1). Here, only the positive equilibrium point will be analysed. The possible positive equilibrium point for model (2) is written as $EQ=(x_1, y_1, z_1, w_1)$, where $x_1=\dfrac{k_1+q_2E_2}{\alpha_1\beta_1}$, $y_1=\dfrac{k_2}{\alpha_2\beta_2}$, $z_1=\dfrac{rx_1K-rx_1^{\,2}-\tau_1x_1K+\tau_2y_1K-q_1E_1x_1K}{\alpha_1x_1K}$, and $w_1=\dfrac{sy_1L-sy_1^{\,2}+\tau_1x_1L-\tau_2y_1L}{\alpha_2y_1L}$. The equilibrium point $EQ=(x_1, y_1, z_1, w_1)$ become a positive equilibrium point when $rx_1K-rx_1^{\,2}-\tau_1x_1K+\tau_2y_1K-q_1E_1x_1K>0$ and $sy_1L-sy_1^{\,2}+\tau_1x_1L-\tau_2y_1L>0$.

From the first variation of model (2), we get the Jacobian matrix as

$$A=\begin{pmatrix}
r-\dfrac{2rx}{K}-\tau_1-\alpha_1z-q_1E_1 & \tau_2 & -\alpha_1x & 0 \\
\tau_1 & s-\dfrac{2sy}{L}-\tau_2-\alpha_2w & 0 & -\alpha_2y \\
\alpha_1\beta_1z & 0 & \alpha_1\beta_1x-k_1-q_2E_2 & 0 \\
0 & \alpha_2\beta_2w & 0 & \alpha_2\beta_2y-k_2
\end{pmatrix}.$$

After substituting the equilibrium point $EQ=(x_1, y_1, z_1, w_1)$ into the Jacobian matrix A we get

$$A_E=\begin{pmatrix}
d_1 & \tau_2 & -d_2 & 0 \\
\tau_1 & d_3 & 0 & -d_4 \\
d_5 & 0 & d_6 & 0 \\
0 & d_7 & 0 & d_8
\end{pmatrix},$$

where $d_1 = r - \frac{2rx_1}{K} - \tau_1 - \alpha_1 z_1 - q_1 E_1$, $d_2 = \alpha x_1$, $d_3 = s - \frac{2sy_1}{L} - \tau_2 - \alpha_2 w_1$, $d_4 = \alpha_2 y_1$, $d_5 = \alpha_1 \beta_1 z_1$, $d_6 = \alpha_1 \beta_1 x_1 - k_1 - q_2 E_2$, $d_7 = \alpha_2 \beta_2 w_2$, and $d_8 = \alpha_2 \beta_2 y_2 - k_2$.

The characteristic equation associated with the Jacobian matrix A_E is given by $f(\lambda) = \det(\lambda I - A_E)$, i.e. $f(\lambda) = \lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0$, where

$b_3 = -(d_1 + d_3 + d_6 + d_8)$, $b_2 = (d_3 + d_6 + d_8)d_1 + d_2 d_5 + d_3 d_6 - \tau_1\tau_2 + d_4 d_7 + d_3 d_8 + d_6 d_8$,
$b_1 = (-d_3 d_6 - d_3 d_8 - d_4 d_7 - d_6 d_8)d_1 + (\tau_1\tau_2 - d_3 d_8 - d_4 d_7)d_6 + (\tau_1\tau_2 - d_2 d_5)d_8 - d_2 d_3 d_5$,
and $b_0 = (d_3 d_6 d_8 + d_4 d_6 d_7)d_1 + d_2 d_4 d_5 d_7 - \tau_1\tau_2 d_6 d_8 + d_2 d_3 d_5 d_8$.

The equilibrium point $EQ = (x_1, y_1, z_1, w_1)$ is locally asymptotically stable when the Routh-Hurwitz stability test [16], i.e. $b_0 > 0$, $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_3 b_2 b_1 - b_1^2 - b_0 b_3^2 > 0$ are satisfied.

Example 1. Suppose the parameter values for model (1) are given as $r = 1.5$, $s = 1.5$, $\tau_1 = 0.25$, $\tau_2 = 0.50$, $\beta_1 = 0.0511$, $\beta_2 = 0.0512$, $K = 1000$, $\alpha_1 = 0.0521$, $\alpha_2 = 0.0522$, $L = 1000$, $k_1 = 0.51$, and $k_2 = 0.52$ in appropriate units. Then we have the positive equilibrium point $EQ = (191.56297, 194.56418, 28.22436, 18.28155)$. The characteristic equation associated with Jacobian matrix evaluated at the equilibrium point EQ is $f(\lambda) = \lambda^4 + 1.33316\lambda^3 + 1.54898\lambda^2 + 0.79806\lambda + 0.37215$. From the characteristic equation, it is easy to check that the Routh-Hurwitz criterion follows and the eigen values are $-0.50382 \pm 0.6163i$ and $-0.16277 \pm 0.74886i$. All of real parts of the eigen values are negative, then the equilibrium point EQ is locally asymptotically stable. This means the two preys and the two predators will be sustainable for a long period of time.

3. Optimal harvesting policies

The stable positive equilibrium point EQ of the model (2) is associated with the maximum profit problem. Under consideration that both prey and predator populations in the unprotected area for fishing are economically valuable, then the populations in that area are harvested with constant efforts. Exploitation activities require cost function and gives revenue consequences. Then we define the total cost function as $TC = cE$, where c denotes the unit cost of exploitation and E is the constant harvesting effort. While the total revenue function is denoted as $TR = pY(E)$, where p is the unit price of population stock (N). The yield of exploitation is then denoted by $Y(E, N) = qEN$, where q is the cathability coefficient of the population. Therefore we get the profit function as $\pi = TR - TC$. Since the positive equilibrium point $EQ = (x_1, y_1, z_1, w_1)$ depends on the efforts then the profit function depends also on the efforts and then it is written as $\pi(E) = TR(E) - TC(E)$.

In order to get the equilibrium point $EQ = (x_1, y_1, z_1, w_1)$ become a positive point, the values of harvesting efforts E_1 and E_2 have to follow the conditions (i) $rx_1K - rx_1{}^2 - \tau_1 x_1 K + \tau_2 y_1 K - q_1 E_1 x_1 K > 0$ and (ii) $sy_1 L - sy_1{}^2 + \tau_1 x_1 L - \tau_2 y_1 L > 0$. Condition (i) is written as

$$E_1 \le f(E_2) = \frac{1}{-q_1 K \alpha_2 \beta_2 \beta_1 \alpha_1 (k_1 + q_2 E_2)} \Big\{ rk_1^2 \alpha_2 \beta_2 + rq_2^2 E_2^2 \alpha_2 \beta_2 + 2rk_1 \alpha_2 \beta_2 q_2 E_2 - rK\alpha_2 \beta_2 \beta_1 \alpha_1 k_1 - rK\alpha_2 \beta_2 \beta_1 \alpha_1 q_2 E_2 - K\alpha_1^2 \beta_1^2 k_2 \tau_2 + \tau_1 K \alpha_2 \beta_2 \alpha_1 \beta_1 k_1 + \tau_1 K \alpha_2 \beta_2 \alpha_1 \beta_1 q_2 E_2 \Big\}.$$

Condition (ii) can be written as

$$E_2 > E_{2l} = \frac{-\beta_1\alpha_1 k_2 sL\beta_2\alpha_2 + \beta_1\alpha_1 k_2^2 s + \beta_1\alpha_1 k_2\tau_2 L\beta_2\alpha_2 - k_1\tau_1 L\beta_2^2\alpha_2^2}{q_2\tau_1 L\beta_2^2\alpha_2^2}.$$

The equilibrium point EQ becomes a positive point when $(E_1, E_2)\in D$, where $D = \{(E_1, E_2): 0 \le E_1 \le \min\{E_{1\max}, f(E_2)\}, \max\{0, E_{2l}\} \le E_2 \le E_{2\max}\}$.
The profit function from the exploitation of both prey and predator in the unprotected area at the equilibrium point $EQ = (x_1, y_1, z_1, w_1)$ is given by

$$\pi(E_1, E_2) = p_1 q_1 x_1 E_1 + p_2 q_2 z_1 E_2 - (c_1 E_1 + c_2 E_2). \tag{3}$$

The profit function (3) will be maximized in the feasible region of D for some values of $E_{1\max}$ and $E_{2\max}$. The critical values of the efforts E_1 and E_2 are determined by considering the first partial derivatives and the critical values of the efforts at the boundary of D.

The biological equilibrium is determined by evaluating the system of equations $\frac{dx}{dt}=0$, $\frac{dy}{dt}=0$, $\frac{dz}{dt}=0$, and $\frac{dw}{dt}=0$, simultaneously. The economic equilibrium is reached when the total revenue is the same with the total cost. The profit function for the exploited prey and predator populations is written as $\pi(E_1, E_2) = p_1 q_1 x E_1 + p_2 q_2 z E_2 - c_1 E_1 - c_2 E_2$. Our objective is to maximize the present value of net revenues for infinite horizon problem which is given by

$$J = \int_0^\infty e^{-\delta t}\{(p_1 q_1 x - c_1)E_1(t) + (p_2 q_2 z - c_2)E_2(t)\}dt. \tag{4}$$

The symbol δ states the discount rate of the net revenue. Our goal is to maximize the present value of J subject to the system of equations (2) with the help of the Pontryagin's maximum principle, as stated in [17]. The control variables $E_1(t)$ and $E_2(t)$ are subject to the conditions $0 \le E_i(t) \le E_{i\max}$ for $i = 1, 2$.

From the problem and improper integral equation (4) we write the Hamiltonian function as

$$\begin{aligned} H = e^{-\delta t}\{(p_1 q_1 x - c_1)E_1 + (p_2 q_2 z - c_2)E_2\} + \lambda_1\left\{rx - \frac{r}{K}x^2 - \tau_1 x + \tau_2 y - \alpha_1 xz - q_1 E_1 x\right\} \\ + \lambda_2\left\{sy - \frac{s}{L}y^2 + \tau_1 x - \tau_2 y - \alpha_2 yw\right\} + \lambda_3\{\beta_1\alpha_1 xz - k_1 z - q_2 E_2 z\} + \lambda_4\{\beta_2\alpha_2 yw - k_2 w\}, \end{aligned} \tag{5}$$

where the variables $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, and $\lambda_4(t)$ denote the adjoints of the problem. We set $\frac{\partial H}{\partial E_1} = 0$ and $\frac{\partial H}{\partial E_2} = 0$ as the necessary conditions for the control variables E_1 and E_2 to be optimal. From the Hamiltonian equation (5), we have $\frac{\partial H}{\partial E_1} = e^{-\delta t}(p_1 q_1 x - c_1) - \lambda_1 q_1 x = 0$ and $\frac{\partial H}{\partial E_2} = e^{-\delta t}(p_2 q_2 z - c_2) - \lambda_3 q_2 z = 0$. Then we get $\lambda_1 = \frac{e^{-\delta t}(p_1 q_1 x - c_1)}{q_1 x}$ and $\lambda_3 = \frac{e^{-\delta t}(p_2 q_2 z - c_2)}{q_2 z}$.

From the Pontryagin's maximum principle $\dot{\lambda}_1 = -\frac{\partial H}{\partial x}$, $\dot{\lambda}_2 = -\frac{\partial H}{\partial y}$, $\dot{\lambda}_3 = -\frac{\partial H}{\partial z}$, and $\dot{\lambda}_4 = -\frac{\partial H}{\partial w}$ and adjoint variables $\lambda_1(t) = \frac{e^{-\delta t}(p_1 q_1 x - c_1)}{q_1 x}$ and $\lambda_3(t) = \frac{e^{-\delta t}(p_2 q_2 z - c_2)}{q_2 z}$ then solving them to find the remains adjoint variables $\lambda_2(t)$ and $\lambda_4(t)$. The adjoint variables $\lambda_2(t)$ and $\lambda_4(t)$ are determined by solving the system of differential equations $\dot{\lambda}_2+\left(s-\frac{2sy}{L}-\tau_2-\alpha_2 w\right)\lambda_2+\beta_2\alpha_2 w\lambda_4=-\tau_2\lambda_1$ and $\dot{\lambda}_4+(\beta_2\alpha_2 y-k_2)\lambda_4-\alpha_2 y\lambda_2=0$ which satisfy the transversality conditions $\lambda_2(t)=0$ and $\lambda_4(t)=0$, as t tends to infinity.

Substitute $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, and $\lambda_4(t)$ into the system of equations $\dot{\lambda}_1=-\frac{\partial H}{\partial x}$ and $\dot{\lambda}_3=-\frac{\partial H}{\partial z}$ and solving them to get E_1 and E_2. Now, the control variables E_1 and E_2 still depend on x, y, z, and w, i.e., $E_1=E_1(x,y,z,w)$ and $E_2=E_2(x,y,z,w)$. By substituting $x=x_1=\frac{k_1+q_2E_2}{\alpha_1\beta_1}$, $y=y_1=\frac{k_2}{\alpha_2\beta_2}$, $z=z_1=\frac{rx_1K-rx_1^2-\tau_1x_1K+\tau_2y_1K-q_1E_1x_1K}{\alpha_1x_1K}$, and $w=w_1=\frac{sy_1L-sy_1^2+\tau_1x_1L-\tau_2y_1L}{\alpha_2y_1L}$ we possibly get the suitable values of control variables E_1 and E_2. The values of E_1, E_2, x_1, and z_1 maximize the present value of revenues J.

Example 2. Suppose the parameter values for model (2) are given as $r=3.5$, $s=3.5$, $\tau_1=0.25$, $\tau_2=0.50$, $\beta_1=0.1511$, $\beta_2=0.1512$, $K=100000$, $\alpha_1=0.1221$, $\alpha_2=0.1222$, $L=1000$, $k_1=0.61$, $k_2=0.62$, $q_1=0.5$, and $q_2=0.5$ in appropriate units. Take $p_1=10$, $p_2=12$, $c_1=5$, and $c_2=6$. Then we get equilibrium point $EQ=(x_1,y_1,z_1,w_1)$, where $x_1=33.06357+27.10128E_2$, $y_1=33.55585$,

$$z_1=\frac{7.7205\cdot10^{16}+5.8678\cdot10^{16}E_2-1.7138\cdot10^{13}E_2^2-1.1021\cdot10^{16}E_1-9.0338\cdot10^{13}E_1E_2}{4.5132\cdot10^{16}+3.6993\cdot10^{16}E_2},$$ and

$w_1=28.60195+1.652304E_2$.

Suppose that $E_{1\max}=1$ and $E_{2\max}=1$, then we get the feasible region of $D=\{(E_1,E_2):0\le E_1\le\min\{1,f(E_2)\},\max\{0,E_{2l}\}\le E_2\le1\}$. After doing a few calculations we get the value of E_{2l} as $E_{2l}=-17.31034$ and $f(E_2)=\frac{0.02600(-62181E_2^2+2.12899\cdot10^8E_2+2.80121\cdot10^8)}{8.52203\cdot10^5E_2+1.039688\cdot10^6}$. It is easy to check that $f(E_2)\ge6.77465$ for $E_2\in[0,1]$. Then the region of D becomes $D=\{(E_1,E_2):0\le E_1\le1,0\le E_2\le1\}$. The profit function is now written as

$$\pi(E_1,E_2)=\frac{5.000\cdot10^{-10}}{2.018\cdot10^6+1.654\cdot10^6E_2}\Big\{6.472\cdot10^{17}E_1+9.783\cdot10^{17}E_1E_2+3.671E_1E_2^2+6.706\cdot10^{17}E_2+5.082\cdot10^{17}E_2^2-1.542\cdot10^{14}E_2^3\Big\}.$$

The critical values of the efforts E_1 and E_2 are determined by considering the first partial derivatives and the values at the boundary of D and we get a pair of the critical value of efforts

$(E_1^*, E_2^*) = (1, 1)$ that maximizes the profit function with the value of $\pi(1,1) = 431.7077$. The value of efforts $E_1 = 1$ and $E_2 = 1$ are at the maximum level. The value pairs of the efforts lies at the boundary of D. The critical value $(E_1^*, E_2^*) = (1, 1)$ gives a positive equilibrium point $EQ = (60.165, 33.556, 23.647, 30.254)$. The eigen values associated with the positive equilibrium point are $-0.03487 \pm 1.75043i$ and $-0.26061 \pm 1.52602i$. This means that the equilibrium point EQ is locally asymptotically stable. In this situation, the populations will coexist for a long period of time although the prey and the predator populations in the unprotected area for fishing are harvested at the maximum level of efforts.

Example 3. Suppose the parameter values for model (2) are given as $r = 1.5$, $s = 1.5$, $\tau_1 = 0.25$, $\tau_2 = 0.50$, $\beta_1 = 0.0511$, $\beta_2 = 0.0512$, $K = 1000$, $\alpha_1 = 0.0521$, $\alpha_2 = 0.0522$, $L = 1000$, $k_1 = 0.51$, $k_2 = 0.52$, $q_1 = 0.8$, and $q_2 = 0.5$ in appropriate units. Take $p_1 = 10$, $p_2 = 12$, $c_1 = 5$, $c_2 = 6$, and $\delta = 0.005$ in appropriate units. Then we have equilibrium point $EQ = (x_1, y_1, z_1, w_1)$, where $x_1 = 191.56296 + 186.80683E_2$, $y_1 = 194.56418$, $z_1 = \dfrac{0.01919(1250x_1 + 1.5x_1^2 - 97282.08810 + 800x_1E_1)}{x_1}$, and $w_1 = 18.28156 + 4.62294E_2$.

With the help of the Pontryagin's maximum principle and by considering the transversality condition for the infinite horizon problem we get $E_1 = 1.07984$ and $E_2 = 1.18462$. Then the positive equilibrium point is $EQ = (414.0431, 194.5642, 0.0003, 23.7580)$ with the eigen values -1.00773, $-0.33606 \pm 0.65943i$, and -0.00002. Under these conditions, the positive equilibrium point EQ is locally asymptotically stable. The adjoint variables are written as $\lambda_1 = 9.98490e^{-0.005t}$, $\lambda_2 = -0.03846\, e^{-0.005t}$, $\lambda_3 = -41653.55154\, e^{-0.005t}$, and $\lambda_4 = 78.12952\, e^{-0.005t}$. Then we get the maximum present value of the revenues $J = \int_0^\infty 3564.317259\, e^{-0.005t}\, dt = 7.128634518 \cdot 10^5$.

4. Conclusions

We have studied the dynamical behavior of prey and predator in a two patch environment. The model for dynamics of prey and predator without harvesting in the protected area for fishing as well as in the unprotected area for fishing possibly has a positive equilibrium point. Under certain conditions of parameter values, the equilibrium point is locally asymptotically stable which means that the prey and predator populations can live in coexistence for a long period of time.

For the model with harvesting of prey and predator in the unprotected area for fishing, the stable equilibrium point is related to the problems of maximizing profit and present value of revenues. When the prey and predator are harvested at the maximum level of the efforts, the exploitation activity maximizes the profit function, but over exploitation will lead to the extinction for harvested predator. For the problem of maximizing the present value of revenues, there exists the extremal for the efforts maximizing the present value of revenues. Beside that, the prey and predator populations will remain coexistence for a long period of time.

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