Optimal harvesting policy of predator-prey model with free fishing and reserve zones

Rustam Rustam
Telkom University

Optimal harvesting policy of predator-prey model with free fishing and reserve zones

Syamsuddin Toaha and Rustam

Citation: AIP Conference Proceedings **1825**, 020023 (2017); doi: 10.1063/1.4978992
View online: http://dx.doi.org/10.1063/1.4978992
View Table of Contents: http://aip.scitation.org/toc/apc/1825/1
Published by the American Institute of Physics

Optimal Harvesting Policy of Predator-Prey Model with Free Fishing and Reserve Zones

Syamsuddin Toaha[1,a)] and Rustam[2, b)]

[1]*Department of Mathematics, Hasanuddin University,*
Jln. Perintis Kemerdekaan, KM 10, 90245,
Makassar, Indonesia
[2]*Department of Mathematics, Universitas Sembilanbelas November,*
Jln. Pemuda, Kolaka, Indonesia

[a)]syamsuddint@yahoo.com
[b)]rustam.math06@gmail.com

Abstract. The present paper deals with an optimal harvesting of predator-prey model in an ecosystem that consists of two zones, namely the free fishing and prohibited zones. The dynamics of prey population in the ecosystem can migrate from the free fishing to the prohibited zone and vice versa. The predator and prey populations in the free fishing zone are then harvested with constant efforts. The existence of the interior equilibrium point is analyzed and its stability is determined using Routh-Hurwitz stability test. The stable interior equilibrium point is then related to the problem of maximum profit and the problem of present value of net revenue. We follow the Pontryagin's maximal principle to get the optimal harvesting policy of the present value of the net revenue. From the analysis, we found a critical point of the efforts that makes maximum profit. There also exists certain conditions of the efforts that makes the present value of net revenue becomes maximal. In addition, the interior equilibrium point is locally asymptotically stable which means that the optimal harvesting is reached and the unharvested prey, harvested prey, and harvested predator populations remain sustainable. Numerical examples are given to verify the analytical results.

INTRODUCTION

The dynamics of predator and prey populations which consists of two populations or three populations is one of the topics of research in mathematical ecology which has been widely studied. Some authors considered two preys and one predator or one prey and two predators in the population dynamics. The other researchers studied two stages, mature and immature in the predator or in the prey, some others reseachers considered two zones of the ecosystem, namely a free fishing zone and the other is a prohibited zone where the predator or prey migrates between the two zones. In [1] the dynamics of a fishery resources in an aquatic ecosystem which consists of a free fishing zone and a prohibited zone was considered and the fish population was protected from over exploitation by controlling the tax. The system of predator-prey with two stages structure for predator and prey in a two parts of ecosystem with harvesting for mature predator and mature prey in the free fishing zone was anayzed by [2] and the analysis showed that there exists certain conditions to get the optimal harvesting.

The dynamics of predator and prey populations with selective harvesting has been deeply studied in many perspectives. Some authors considered harvesting for the prey population only (for examples in [1, 3, 4, 5]), harvesting for the predator population only (for examples in [6, 7, 8, 9, 10, 11, 12]), and harvesting for both predator and prey populations (for examples in [13, 14]). Most of predator-prey model with harvesting was related to the economic problems including maximum profit problem, taxation effect, and total discounted net revenue problem, for examples in [1, 14, 15].

Based on the previous results of the authors, we investigate the dynamics of predator and prey populations with prey can migrate between the two zones, one of which is a free fishing zone and the other is a prohibited fishing

Symposium on Biomathematics (SYMOMATH 2016)
AIP Conf. Proc. 1825, 020023-1–020023-8; doi: 10.1063/1.4978992
Published by AIP Publishing. 978-0-7354-1493-8/$30.00

zone. The predator and prey populations in the free fishing zone are then exploited with constant efforts. We analyze the existence and the stability of the interior equilibrium point. The stable interior equilibrium point is then related to the problem of maximum profit and to the problem of maximum present value of net revenue. We determine the critical point of the efforts that makes the profit function becomes maximum and also the present value of net revenue becomes maximum. The Routh-Hurwitz stability tes is applied to decide the kind of the stability of the interior equilibrium point. The Pontryagin's maximal principle is applied to find out the optimal harvesting policy of present value of net revenue.

DYNAMICS OF PREDATOR AND PREY WITH PROHIBITED ZONE

We consider a predator-prey fishery management in an ecosystem which consists of two zones, namely a free fishing zone and a prohibited zone, where no fishing is allowed in this zone. The two zones are supposed have the same characteristics, and the prey population can migrate in the two zones freely. The growth of the prey in the free fishing zone and in the prohibited zone in the absence of predator is assumed to be logistic. We assume that the predator just cathes the prey in the free fishing zone. The dynamics of predator and prey populations is denoted as a system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-a_1x+a_2y-a_3x-\alpha xz\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+a_1x-a_2y-a_4y\\ \frac{dz}{dt} &= m\alpha xz-kz.\end{aligned} \tag{1}$$

The symbols $x=x(t)$ and $y=y(t)$ state the size of prey population in the free fishing zone and in the prohibited zone at time *t*, respectively. The symbol $z=z(t)$ states the size of the predator population in the free fishing zone at time *t*. Parameter r is the intrinsic growth rate of the prey population in the free fishing zone. Parameter *K* is carrying capacity of the ecosystem for the prey population. The level of predation is expressed by the parameter α, and the value of m $(0<m<1)$ is the scaled of predation to the predator. Parameter s is the intrinsic growth of prey population in reserve zone, L is the carrying capacity of the ecosystem for the prey population in the prohibited zone. Parameters a_3, a_4, and *k* are the mortality rate for the prey in free fishing zone, prey in prohibited zone, and predator respectively. Parameter a_1 states the migration rate from the prey in the free fishing zone to the prey in prohibited zone, while parameter a_2 states the migration rate from the prey population in the prohibited zone to the prey population in the free fishing zone.

By considering that the populations are economically valuable, then the predator and the prey populations in free fishing zone are exploited with constant efforts. The dynamics of predator and prey populations is then extended and written in the form

$$\begin{aligned}\frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-a_1x+a_2y-a_3x-\alpha xz-q_1E_1x\\ \frac{dy}{dt} &= sy\left(1-\frac{y}{L}\right)+a_1x-a_2y-a_4y\\ \frac{dz}{dt} &= m\alpha xz-kz-q_3E_3z.\end{aligned} \tag{2}$$

In the model (2), parameters q_1 and q_3 state the cathability coefficient for the prey and the predator populations respectively. Parameters E_1 and E_3 state the harvesting efforts satisfying $0\le E_i\le E_{i\max}$ for $i=1,3$ and some value of $E_{i\max}$. Let $r_1=r/K$, $r_2=r-a_1-a_3$, $s_1=s/L$, $s_2=s-a_2-a_4$, and $a_5=m\alpha$, then model (2) becomes

$$\begin{aligned}\frac{dx}{dt} &= r_2 x - r_1 x^2 + a_2 y - \alpha xz - q_1 E_1 x \\ \frac{dy}{dt} &= s_2 y - s_1 y^2 + a_1 x \\ \frac{dz}{dt} &= a_5 xz - kz - q_3 E_3 z.\end{aligned} \tag{3}$$

From the model (3), we have three non negative equilibrium points, namely $T_1 = (0, 0, 0)$, $T_2 = (x_2, y_2, 0)$, and $T_3 = (x_3, y_3, z_3)$, where $x_3 = \frac{q_3 E_3 + k}{a_5}$, $y_3 = \frac{s_2 + \sqrt{s_2^2 + 4 s_1 a_1 x_3}}{2 s_1}$, and $z_3 = \frac{r_2 x_3 - r_1 x_3^2 + a_2 y_3 - q_1 E_1 x_3}{\alpha x_3}$. When $r_2 x_3 - r_1 x_3^2 + a_2 y_3 - q_1 E_1 x_3 > 0$, the equilibrium point T_3 becomes an interior equilibrium point. For analyzes we use the Jacobian matrix to get the linear model and by evaluating the Jacobian matrix at the equilibrium point $T_3 = (x_3, y_3, z_3)$, we have

$$J_E = \begin{pmatrix} r_2 - 2r_1 x_3 - \alpha z_3 - q_1 E_1 & a_2 & -\alpha x_3 \\ a_1 & s_2 - 2 s_1 y_3 & 0 \\ a_5 z_3 & 0 & a_5 x_3 - k - q_3 E_3 \end{pmatrix}.$$

The polynomial characteristic associated with the Jacobian matrix J_E is given by $f(\lambda) = \det(\lambda I - J_E)$, i.e $f(\lambda) = \lambda^3 + b_2 \lambda^2 + b_1 \lambda + b_0$, where

$$\begin{aligned}
b_2 &= \alpha z_1 + q_3 E_3 - s_2 - r_2 + q_1 E_1 + k - a_5 x_3 + 2 s_1 y_3 + 2 r_1 x_3, \\
b_1 &= -2 s_1 y_3 a_5 x_3 + 2 s_1 y_3 q_3 E_3 + 2 r_1 x_3 q_3 E_3 + \alpha z_3 q_3 E_3 - q_1 E_1 a_5 x_3 + q_1 E_1 q_3 E_3 - 2 r_2 s_1 y_3 \\
&\quad - 2 r_1 x_3 s_2 - \alpha z_1 s_2 - q_1 E_1 s_2 - a_1 a_2 + r_2 s_2 + 2 r_1 x_3 k + \alpha z_1 k + r_2 a_5 x_3 + q_1 E_1 k + 2 s_1 y_1 k \\
&\quad - r_2 q_3 E_3 - 2 r_1 a_5 x_3^2 - s_2 q_3 E_3 + s_2 a_5 x_3 - r_2 k - s_2 k + 4 r_1 x_3 s_1 y_3 + 2 \alpha z_3 s_1 y_3 + 2 q_1 E_1 s_1 y_3, \\
b_0 &= 4 r_1 x_3 s_1 y_3 q_3 E_3 + 2 \alpha z_3 s_1 y_3 q_3 E_3 - 2 q_1 E_1 s_1 y_3 a_5 x_3 + 2 q_1 E_1 s_1 y_3 q_3 E_3 + r_2 s_2 k - a_1 a_2 k \\
&\quad + 2 r_2 s_1 y_3 a_5 x_3 - 2 r_2 s_1 y_3 q_3 E_3 - 2 r_1 s_2 x_3 q_3 E_3 - 4 r_1 s_1 a_5 y_3 x_3^2 + 4 r_1 x_3 s_1 y_3 k - \alpha z_3 s_2 q_3 E_3 \\
&\quad + 2 \alpha z_3 s_1 y_3 k + q_1 E_1 s_2 a_5 x_3 - q_1 E_1 s_2 q_3 E_3 + 2 q_1 E_1 s_1 y_3 k - r_2 s_2 a_5 x_3 + r_2 s_2 q_3 E_3 - 2 r_2 s_1 y_3 k \\
&\quad + 2 r_1 s_2 a_5 x_3^2 - 2 r_1 x_3 s_2 k - \alpha z_3 s_2 k - q_1 E_1 s_2 k + a_1 a_2 a_5 x_3 - a_1 a_2 q_3 E_3.
\end{aligned}$$

Referring to the Routh-Hurwitz criterion [16], the interior equilibrium point T_3 is locally asymptotically stable when the conditions $b_0 > 0$, $b_2 > 0$, and $b_2 b_1 - b_0 > 0$ are satisfied.

BIOLOGICAL AND ECONOMIC EQUILIBRIUM

The biological and economic equilibrium is a notion that integrates biological aspect and economical aspect in the population dynamics, as considered in [17]. The biological equilibrium means that there is not changes of the population, which is found by solving the equations $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$, and $\frac{dz}{dt} = 0$ simultaneously. The economic equilibrium means that the total revenue from exploitation of populations equals to the total cost of exploitation efforts. We just consider that the total cost is proportional to the exploitation effort, written as $TC = C(E) = cE$. We also suppose that the total revenue from the exploitation of population is proportional to the exploitation yield, written as $TR = pY(E)$, where p is the unit price of the stock and $Y(E) = Eqx$ denotes the exploitation yield function which has been studied in [18]. The profit function is then given by $\pi = TR - TC$. Let c_i states exploitation cost per unit effort of population x_i and p_i denoted the price per unit biomass of population x_i. The profit function of exploitation for predator and prey populations in the free fishing zone is given by

$\pi = (p_1 q_1 x)E_1 + (p_3 q_3 z)E_3 - (c_1 E_1 + c_3 E_3)$. The bionomic equilibrium $(x^*, y^*, z^*, E_1^*, E_3^*)$ is found by solving the following simultaneous equation

$$r_2 x - r_1 x^2 + a_2 y - \alpha xz - q_1 E_1 x = 0$$
$$s_2 y - s_1 y^2 + a_1 x = 0$$
$$a_5 xz - kz - q_3 E_3 z = 0$$
$$(p_1 q_1 x)E_1 + (p_3 q_3 z)E_3 - (c_1 E_1 + c_3 E_3) = 0.$$

We then associate the interior equilibrium point T_3 to the maximum profit problem. The point T_3 is an interior equilibrium point if the conditions $0 \le E_i \le E_{i\max}$ for $i = 1, 3$ and

$$r_2 x_3 - r_1 x_3^2 + a_2 y_3 - q_1 E_1 x_3 > 0 \tag{4}$$

are satisfied.

The profit function at the equilibrium point T_3 is written in the form

$$\pi(E_1, E_3) = (p_1 q_1 x_3)E_1 + (p_3 q_3 z_3)E_3 - (c_1 E_1 + c_3 E_3).$$

The problem is determining a pair of efforts (E_1, E_3) which satisfies (4) and $0 \le E_i \le E_{i\max}$ for $i = 1, 3$ that maximizes the profit function $\pi(E_1, E_3)$. We also need the interior equilibrium point T_3 is always asymptotically stable.

OPTIMAL HARVESTING POLICY OF THE POPULATION

The biological equilibrium is found by evaluating the equations $\frac{dx}{dt} = 0$, $\frac{dy}{dt} = 0$, and $\frac{dz}{dt} = 0$ simultaneously. The economic equilibrium is found when the total revenue equals the total cost. The profit function for the exploited predator and prey populations is written as $\pi(E_1, E_3) = p_1 q_1 x E_1 + p_3 q_3 z E_3 - c_1 E_1 - c_3 E_3$. Our goal is to maximize the present value *J* of net revenue function which is given by

$$J = \int_0^\infty e^{-\delta t}\{(p_1 q_1 x - c_1)E_1(t) + (p_3 q_3 z - c_3)E_3(t)\}dt. \tag{5}$$

The symbol δ states the discount rate of the net revenue. Our goal is to maximize the present value *J* subject to the equation (3) by following the Pontryagin's maximum principle [19]. Variables $E_1(t)$ and $E_3(t)$ as a control are subject to the condition $0 \le E_i(t) \le E_{i\max}$ for $i = 1, 3$.

From this problem we write the Hamiltonian function as

$$\begin{aligned} H = e^{-\delta t}\{(p_1 q_1 x - c_1)E_1 + (p_3 q_3 z - c_3)E_3\} + \lambda_1\{r_2 x - r_1 x^2 + a_2 y - \alpha xz - q_1 E_1 x\} \\ + \lambda_2\{s_2 y - s_1 y^2 + a_1 x\} + \lambda_3\{a_5 xz - kz - q_3 E_3 z_3\}, \end{aligned} \tag{6}$$

where the variables $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ state the adjoints of the problem.

We set $\frac{\partial H}{\partial E_1} = 0$ and $\frac{\partial H}{\partial E_3} = 0$ as the necessary conditions for the control variables E_1 and E_3 to be optimal. From the Hamiltonian function (6), we have $\frac{\partial H}{\partial E_1} = e^{-\delta t}(p_1 q_1 x - c_1) - \lambda_1 q_1 x = 0$ and $\frac{\partial H}{\partial E_3} = e^{-\delta t}(p_3 q_3 z - c_3) - \lambda_3 q_3 z = 0$.

Then we get $\lambda_1 = \frac{e^{-\delta t}(p_1 q_1 x - c_1)}{q_1 x}$ and $\lambda_3 = \frac{e^{-\delta t}(p_3 q_3 z - c_3)}{q_3 z}$. From the Hamiltonian equation we also have

$$\frac{\partial H}{\partial x} = e^{-\delta t} p_1 q_1 E_1 + \lambda_1(r_2 - 2r_1 x - \alpha z - q_1 E_1) + \lambda_2 a_1 + \lambda_3 a_5 z,$$

$$\frac{\partial H}{\partial y} = +\lambda_1 a_2 + \lambda_2(s_2 - 2s_1 y), \text{ and}$$

$$\frac{\partial H}{\partial z} = e^{-\delta t} p_3 q_3 E_3 - \lambda_1 \alpha x + \lambda_3 (a_5 x - k - q_3 E_3).$$

From the Pontryagin's maximum principle $\dot{\lambda}_1 = -\frac{\partial H}{\partial x}$, $\dot{\lambda}_2 = -\frac{\partial H}{\partial y}$, and $\dot{\lambda}_3 = -\frac{\partial H}{\partial z}$ we get

$$\frac{\delta e^{-\delta t}(-p_1 q_1 x + c_1)}{q_1 x} + e^{-\delta t} p_1 q_1 E_1 + \lambda_1 (r_2 - 2r_1 x - \alpha z - q_1 E_1) + \lambda_2 a_1 + \lambda_3 a_5 z = 0, \tag{7}$$

$$\dot{\lambda}_2 + (s_2 - 2s_1 y)\lambda_2 + a_2 \lambda_1 = 0, \text{ and} \tag{8}$$

$$\frac{\delta e^{-\delta t}(-p_3 q_3 z + c_3)}{q_3 z} + e^{-\delta t} p_3 q_3 E_3 - \lambda_1 \alpha x + \lambda_3 (a_5 x - k - q_3 E_3) = 0. \tag{9}$$

By substituting $\lambda_1 = \frac{e^{-\delta t}(p_1 q_1 x - c_1)}{q_1 x}$ into (8) and considering transversality condition $\lambda_2(t) = 0$ as $t \to \infty$, we get $\lambda_2 = \frac{e^{-\delta t} a_2 (-p_1 q_1 x + c_1)}{q_1 x(-\delta + s_2 - 2s_1 y)}$. Again, substituting $\lambda_1 = \frac{e^{-\delta t}(p_1 q_1 x - c_1)}{q_1 x}$, $\lambda_2 = \frac{e^{-\delta t} a_2 (-p_1 q_1 x + c_1)}{q_1 x(-\delta + s_2 - 2s_1 y)}$, and $\lambda_3 = \frac{e^{-\delta t}(p_3 q_3 z - c_3)}{q_3 z}$ into equations (7) and (9) we get E_1 and E_3, where

$$\begin{aligned} E_1 = \frac{1}{q_1 q_3 c_1 (\delta - s_2 + 2s_1 y)} \{ & 2\alpha p_1 q_1 q_3 s_1 xyz - \delta p_1 q_1 q_3 s_2 x - \delta r_2 p_1 q_1 q_3 x + r_2 p_1 q_1 q_3 s_2 x + 2\delta r_1 p_1 q_1 q_3 x^2 \\ & - 2r_1 s_2 p_1 q_1 q_3 x^2 - 4r_1 q_3 c_1 s_1 xy - 2\alpha c_1 s_1 q_3 yz - a_1 a_2 p_1 q_1 q_3 x + 2a_5 q_1 c_3 s_1 xy + p_1 q_1 q_3 x \delta^2 - 2\delta c_1 q_3 s_1 y \\ & + 2r_2 q_3 c_1 s_1 y - 2x r_1 q_3 c_1 \delta + 2x r_1 q_3 c_1 s_2 - \alpha q_3 z c_1 \delta + \alpha q_3 z c_1 s_2 + a_5 x q_1 c_3 \delta - a_5 x q_1 c_3 s_2 + \delta c_1 q_3 s_2 \\ & + r_2 q_3 c_1 \delta - r_2 q_3 c_1 s_2 - 2a_5 x q_1 p_3 q_3 s_1 y^2 + 2\delta p_1 q_1 x q_3 s_1 y + \alpha q_3 z p_1 q_1 x \delta + 4r_1 q_3 p_1 q_1 s_1 y x^2 \\ & - 2r_2 q_3 p_1 q_1 s_1 xy - \alpha q_3 z p_1 q_1 x s_2 + a_1 a_2 q_3 c_1 + a_5 x q_1 p_3 q_3 y s_2 - a_5 x q_1 p_3 q_3 y \delta - c_1 q_3 \delta^2 \}, \end{aligned} \tag{10}$$

$$\begin{aligned} E_3 = \frac{1}{q_1 q_3 (p_3 q_3 z - p_3 q_3 y + c_3)} \{ & \delta p_3 q_1 q_3 z - \delta c_3 q_1 + a_5 q_1 c_3 x + \alpha p_1 q_1 q_3 xz - \alpha c_1 q_3 z \\ & - a_5 p_3 q_1 q_3 xy + k p_3 q_1 q_3 y - k q_1 c3 \}. \end{aligned} \tag{11}$$

By substituting $x = x_3 = \frac{q_3 E_3 + k}{a_5}$, $y = y_3 = \frac{s_2 + \sqrt{s_2^2 + 4s_1 a_1 x_3}}{2s_1}$, and $z = z_3 = \frac{r_2 x_3 - r_1 x_3^2 + a_2 y_3 - q_1 E_1 x_3}{\alpha x_3}$ into equations (10) and (11) we get the value of control variables E_1 and E_3. The values of E_1, E_3, x_3, y_3, and z_3 maximize the present value of *J*.

NUMERICAL EXAMPLES

Based on the analytical results above, we give some examples to simulate the problem of maximum profit and the problem of maximizing the present value of net revenue.

For the problem of maximum profit function, we set the parameter values as $r = 1.5$, $a_1 = 0.0018$, $a_2 = 0.0175$, $a_3 = 0.0001$, $a_4 = 0.0001$ $K = 100000$, $\alpha = 0.15$, $s = 1.4$, $L = 110000$, $m = 0.112$, $k = 0.0001$, $q_1 = 4.1$, and $q_3 = 4.5$ in appropriate units. Take $p_1 = 25$, $p_3 = 15$, $c_1 = 1000$, and $c_3 = 1020$ in appropriate units. Then we have the equilibrium point $T_3 = (x_3, y_3, z_3)$, where

$$x_3 = 0.0059524 + 267.85714 E_3,$$

$$y_3 = 54308.571 + 39285.714\sqrt{1.9110298 + 0.0000245 E_3}, \text{ and}$$

$$z_3 = \frac{1}{0.0008929 + 40.178571 E_3} \{-0.0001500\,(0.0059524 + 267.85714 E_3)^2 + 950.40892 + 401.27679 E_3 + 687.49999 \sqrt{1.9110298 + 0.0000245 E_3} - 4.1 E_1 (0.0059524 + 267.85714 E_3)\}.$$

In order for equilibrium point T_3 to be an interior equilibrium point, the exploitation efforts E_1 and E_3 must satisfy the conditions $P_3 > 0$ where

$$P_3 = \{-0.0001500\,(0.0059524 + 267.85714 E_3)^2 + 950.40892 + 401.27679 E_3 + 687.49999 \sqrt{1.9110298 + 0.0000245 E_3} - 4.1 E_1 (0.0059524 + 267.85714 E_3)\}$$

and $0 \le E_i(t) \le E_{i\max}$ for $i = 1, 3$. We let $E_{1\max} = 1$ and $E_{3\max} = 1$. In the other words, the equilibrium point T_3 becomes an interior point when $(E_1, E_3) \in D_1$, where $D_1 = \{(E_1, E_3) : 0 \le E_1 \le 1, 0 \le E_3 \le 1\}$.

The profit function associates with the equilibrium point T_3 is given by

$\pi(E_1, E_3) = (p_1 q_1 x_3) E_1 + (p_3 q_3 z) E_3 - (c_1 E_1 + c_3 E_3)$. After substituting the values of x_3, y_3, z_3 and simplifying we get

$$\pi(E_1, E_3) = \{-999.38988 + 27455.357 E_3\} E_1 + \{63132.602 + 27086.183 E_3 - 0.0010125\,(0.0059524 + 267.85714 E_3)^2 + 46406.249 \sqrt{1.9110298 + 0.0000245 E_3} - 276.75 E_1 (0.0059524 + 267.85714 E_3)\} E_3.$$

We have a stationary point $(E_1^*, E_3^*) = (0.0116921, 0.0390229)$ in D_1 and the three critical points $(E_1^*, E_3^*) = (0, 0)$, $(E_1^*, E_3^*) = (0, 1)$, and $(E_1^*, E_3^*) = (1, 1)$ at the boundary of D_1. Unfortunately, the critical point $(E_1^*, E_3^*) = (0.0116921, 0.0390229)$ which gives $\pi(E_1^*, E_3^*) = 3178.0279$ is not a maximum value. The critical point $(E_1^*, E_3^*) = (0, 1)$ maximizes the profit function with the value of $\pi(0,1) = 154298.55$. In this case, there is not harvesting for the prey in the free fishing zone while the predator is harvested at maximum level of effort. By applying the value of harvesting efforts $(E_1^*, E_3^*) = (0, 1)$ we get the equilibrium point $T_3 = (267.86309, 1086174.9, 57.268448)$. The polynomial characteristic of the Jacobian matrix is given by $f(\lambda) = \lambda^3 + 8.4826119 \lambda^2 + 48.472413 \lambda + 53.439865$ which has the eigenvalues -1.3824103 and $-3.5501008 \pm 5.1042929 i$. Under this condition, if we take the values of efforts $E_1^* = 0$ and $E_3^* = 1$, then the predator and the prey populations in the two zones will sustain for a long period of time and also maximize the profit function.

For the problem of maximizing present value of the net revenue, we set the parameter values as $r_1 = 10.5$, $a_1 = 0.180$, $a_2 = 0.175$, $a_3 = 0.0001$, $a_4 = 0.0001$, $K = 100000$, $\alpha = 0.0045$, $m = 0.0212$, $s = 10.4$, $L = 110000$, $k = 0.0001$, $q_1 = 5.1$, and $q_3 = 9.5$ in appropriate units. Take $p_1 = 10$, $p_3 = 10$, $c_1 = 1000$, $c_3 = 1050$, and $\delta = 0.003$ in appropriate units. Then we have the optimal interior equilibrium point $T_3 = (x_3, y_3, z_3)$, where

$$x_3 = 1.0482180 + 99580.713 E_3,$$

$$y_3 = 54073.990 + 5.5434607 \sqrt{9.5151400 x10^{-7} + 6.1694313 x10^{-8} E_3}, \text{ and}$$

$$z_3 = \frac{222.22222}{1.0482180 + 99580.713 E_3} \{-0.0001050\,(1.0482180 + 99580.713 E_3)^2 + 9473.7658 + 1027662.9 E_3 + 9701056.3 \sqrt{9.5151400 x10^{-7} + 6.1694313 x10^{-8} E_3} - 5.1 E_1 (1.0482180 + 99580.713 E_3)\}.$$

The adjoint variables are

$$\lambda_1 = \frac{-0.1960784\, e^{-0.003t}\left(946.54088 - 5078616.3E_3\right)}{1.0482180 + 99580.713E_3},$$

$$\lambda_2 = \frac{0.0343137\, e^{-0.003t}\left(946.54088 - 5078616.3E_3\right)}{\left(1.0482180 + 99580.713E_3\right)\left(-10482.180\sqrt{9.5151400x10^{-7} + 6.1694313x10^{-8}E_3}\right)},\ \text{and}$$

$$\lambda_3 = \frac{-4.74\cdot 10^{-11}e^{-0.003t}\left(1.28\cdot 10^{15} + 7.89\cdot 10^{9}\sqrt{2.64\cdot 10^{10} + 1.71\cdot 10^{9}E_3}\right)\left(1.05\cdot 10^{9} + 9.96\cdot 10^{13}E3\right)}{-2.4\cdot 10^{14} - 2.6\cdot 10^{16}E_3 + 2.6\cdot 10^{16}E_3{}^2 - 1.5\cdot 10^{9}\sqrt{2.6\cdot 10^{10} + 1.7\cdot 10^{9}E_3} + 1.3\cdot 10^{11}E_1 + 1.3\cdot 10^{16}E_1E_3}.$$

After substituting and solving the related equations we get the harvesting efforts $E_1 = 0.0221549$ and $E_3 = 0.9947320$. Then we get the interior equilibrium point $T_3 = \left(99057.168, 109864.55, 0.0000728\right)$. The eigenvalues associted wit the interior equilibirum point are -10.751232, -10.393349, and $-2.945142\cdot 10^{-7}$. Under this condition, the interior equilibrium point T_3 is locally asymptotically stable. The adjoint variables are written as $\lambda_1 = 9.9980206\, e^{-0.003t}$, $\lambda_2 = 0.1658049\ e^{-0.003t}$, and $\lambda_3 = 1.5099596\cdot 10^{10}\ e^{-0.003t}$. Then, we get the maximum value of present value of the net revenue $J = \int_0^\infty 1.1085806\cdot 10^{5}\ e^{-0.003t}dt = 3.6952688\cdot 10^{7}$.

CONCLUSIONS

The dynamics of predator and prey populations with constant harvesting efforts for the predator and the prey populations in the free fishing zone is possible to have an interior equilibrium point $T_3 = \left(x_3, y_3, z_3\right)$. The interior equilibrium point exists when the values of paramaters and harvesting efforts satisfy the conditions $r_2x_3 - r_1x_3{}^2 + a_2y_3 - q_1E_1x_3 > 0$ and $0 \le E_i \le E_{i\max}$. The interior equilibrium point T_3 is locally asymptotically stable when the certain conditions are satisfied. The existence and the stability of the interior equilibrium point T_3 depends on the values of parameters and harvesting efforts. With the condition $0 \le E_i \le E_{i\max}$, there exists a situation such that the interior equilibrium point T_3 is always locally asymptotically stable and also maximizes the profit function. The predator and the prey populations can live in coexistence for a long period of time, although the predator and the prey populations in the free fishing zone are exploited with constant efforts. In addition, the harvested populations in the free fishing zone also give maximum profit. By following the Pontryagin's maximum principle, we found that there exists a specific value of harvesting efforts for the predator and the prey populations in free fishing zone that maximizes the present value of the net revenue. Beside that, the predator and the prey populations in the two zones will not be extinct.

REFERENCES

1. H.F. Hou, H.M. Jiang and X.Y. Meng. A dynamic model for fishery resource with reserve area and taxation. *Journal of Applied Mathematics*, (2012). http://dx.doi.org/10.1155/2012/794719.
2. M.N. Srinivas, M.A.S. Srinivas, K. Das and N.H. Gazi. Prey-predator fishery model with stage structure in two patchy marine aquatic environment. *Applied Mathematics* **2**, 1405-1416 (2011).
3. K. Chakraborty, M. Chakraborty and T.K. Kar. Bifurcation and control of a bioeconomic model of a prey-predator system with time delay. *Nonlinear Analysis: Hybrid Systems* **5**, 613-625 (2011).
4. R.P. Gupta, M. Banerjee and P. Chandra. Bifurcation analysis and control of Leslie–Gower predator–prey model with Michaelis–Menten type prey-harvesting. *Journal Differential Equations and Dynamical Systems* **22**(3), 339-366 (2012).
5. T.K. Kar. A Dynamic reaction model of a prey-predator system with stage-structure for predator. *Modern Applied Science* **4**(5), 183-195 (2010).
6. G.M.V. Laksmi, M. Gunasekaran and S. Vijaya. Bifurcation analysis of predator model with harvested predator. *International Journal of Engineering Research and Development* **10**(6), 42-51 (2014).

7. C. Liu, Q. Zhang and J. Li. Global stability analysis and optimal control of a harvested ecoepidemiological prey predator model with vaccination and taxation. *Abstract and Applied Analysis.* Article ID 950396, 16 pages. (2013). http://dx.doi.org/10.1155/2013/950396.
8. K. Chakraborty, M Chakraborty and T.K. Kar. Optimal control and bifurcation of a prey-predtor model with stage structure. *Applied Mathematics and Computation* **217**(21), 8778-8792 (2011).
9. Y. Qu and J. Wei. Bifurcation analysis in a predator-prey system with stage stucture and harvesting. *Journal of Franklin Institute* **347**(7), 1096-1113 (2010).
10. K. Chakraborty, S. Jana and T.K. Kar. Global dynamics and bifurcation in a stage structured prey-predator fishery model with harvesting. *Applied Mathematics and Computation* **218**(5), 9271-9290 (2012).
11. C. Liu and W. Tang. The dynamics and control of a harvested differential-algebraic prey-predator model. *International Journal of Information and System Sciences* **7**(1), 103-113 (2011).
12. C. Liu, Q. Zhang and X. Duan. Dynamical behavior in a harvested differential-algebraic prey-predator model with discrete time delay and stage structure. *Journal of the Franklin Institute* **346**(10), 1038-1059 (2009).
13. K. Chakraborty, M. Chakraborty and T.K. Kar. Optimal control of harvest and bifurcation of a prey-predator model with stage structure. *Applied Mathematics and Computation* **217**, 8778-8792 (2011).
14. S. Toaha, J. Kusuma, Khaeruddin and Mawardi. Stabilty analysis and optimal harvesting policy of prey-predator model with stage structure for predator. *Applied Mathematical Sciences* **8**(159), 7923-7934 (2014).
15. K. Chakraborty, M. Chakraborty and T.K. Kar. Optimal control of harvest and bifurcation of prey-predator model with stage structure. *Applied Mathematics and Computation* **217**, 8778-8792 (2011).
16. L.E. Keshet, *Mathematical models in biology*. SIAM, Philadelphia, 2005, pp. 231-234
17. D.K. Bhattacharya and S. Begum. Bionomic equilibrium of two-species system, *Mathematical Biosciences* **135,** 111-127 (1996).
18. C.W. Clark, *Bioeconomic modelling and fisheries management*. John Wiley & Sons, New York, 1985, pp. 11-15
19. D. Grass, J.P. Caulkins, G. Feichtinger, G. Tragler and D.A. Behrens, *Optimal control of nonlinear processes*. Springer-Verlag, Berlin Heidelberg, 2003, pp. 122-138.