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Modification of a steepest descent control for output tracking of some class non-minimum phase nonlinear systems[☆]

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A R T I C L E I N F O

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Steepest descent control

A B S T R A C T

The problem of output tracking for a class of nonlinear systems whose zero dynamics are not necessarily stable is addressed in this paper. To solve the problem, we transform the systems into a normal form which is minimum phase with respect to a virtual output, which is a linear combination of state variables. By applying the modified steepest descent control, the system output track to the desired output.

1. Introduction

In the output tracking theory, the input output linearization is one of the most available methods [1]. If the nonlinear system has a stable zero dynamics, a static control law can be chosen such that the system output track to the desired output [1]. On the other hand, if the zero dynamics is unstable, the nonlinear system is called a non-minimum phase [2]. Thus, the standard input output linearization leads to an unstable closed loop system. Recently, output tracking problems for nonlinear non-minimum phase systems have been intensively investigated. The stable inversion proposed in [3] and [4] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on the imaginary axis. In [5], control design procedure for the output tracking was proposed. The design procedure consists of two steps. In the first step, the standard input output linearization is applied. In the second step, we group an output with the internal dynamics as one subsystem, which is usually nonlinear, and the rest of the output as the other subsystem that is linear, the nonlinear subsystems is linearized about its equilibrium. In [6], the asymptotic output tracking which is a class of causal non-minimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. In [7], J. Naiborhu *et.al* have developed a method to design the input control to track the output of a non-minimum phase nonlinear systems asymptotically. The design of the input control is based on the exact linearization. To perform an exact linearization, the output should be selected such that its relative degree is equal to the dimension of the system. Results on stabilization of non-minimum phase system in the output feedback form have been presented in In [8], [9], [10]. The main idea in [8], [9], [10] is output reconstruction such that becomes minimum phase with respect to a new output. Riccardo Marino and Patrizio Tomei [8] have shown how to design a globally stabilizing dynamic output feedback controller of order $n + 2(\rho - 1)$(n is the system order, ρ is the relative degree) for a class

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of nonlinear non-minimum phase systems. To design a dynamic output feedback controller, then the system is required to be minimum phase with respect to a linear combination of the state variables.

[11] has introduced a dynamic feedback control for the asymptotically stability of the minimum phase nonlinear system where unforced dynamic of the system is globally asymptotically stable. In this paper, we will modify the steepest descent control for output tracking of a class non-minimum phase affine nonlinear system, where the relative degree of the system is not well defined. The modification is the addition of an input artificial of the steepest descent control which is different to the one in [11]. In this paper, we show that the modified steepest descent control can be applied to the unstable unforced system also. To apply the modified steepest descent control, the system output will be redefined such that the system becomes minimum phase with respect to a new output.

2. Problem statement and definitions

Consider SISO affine nonlinear control system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}$ is the control input and $y(t) \in \mathcal{R}$ is the measured output. $f : \mathcal{R}^n \to \mathcal{R}^n$ is a smooth function with $f(0) = 0$, $g : \mathcal{R} \to \mathcal{R}^n$ and $h : \mathcal{R}^n \to \mathcal{R}$ are smooth function. Assume also that $h(0) = 0$.

In the following we write the definition of relative degree of the system (1)–(2) according to Isidori [1],

Definition 1. The nonlinear system (1)–(2) is said to have a **relative degree** r at a point x_0, if

(i) $\frac{\partial y^{(k)}}{\partial x} g(x) = 0$ for all x in a neighborhood of x_0 and $k < r - 1$

(ii) $\frac{\partial y^{(r-1)}}{\partial x} g(x_0) \neq 0$

Definition 1 means that we need to differentiate the output of a system r times to generate an explicit relationship between the output y and input u. Let the relative degree of the system be r. Then we have

$$y^{(r)}(t) = a(x(t)) + b(x(t))u, \tag{3}$$

where

$$a(x(t)) = \frac{\partial y^{(r-1)}}{\partial x} f(x(t)) \text{ and } b(x(t)) = \frac{\partial y^{(r-1)}}{\partial x} g(x(t)).$$

Consider again the system (1)–(2) in normal form

$$\dot{\xi}_k = \xi_{k+1}, \qquad k = 1, \ldots, r-1 \tag{4}$$

$$\dot{\xi}_r = a(\xi, \eta) + b(\xi, \eta)u \tag{5}$$

$$\dot{\eta} = q(\xi, \eta)$$

$$y = \xi_1$$

with the internal dynamics

$$\dot{\eta} = q(\xi, \eta). \tag{6}$$

Let $e_i(t) = \xi_i(t) - y_d^{(i-1)}(t)$, $i = 1, \ldots, r$.

Then

$$\dot{e}_k = e_{k+1}, \qquad k = 1, \ldots, r-1 \tag{7}$$

$$\dot{e}_r = a(e + y_d, \eta) + b(e + y_d, \eta)u - y_d^{(r)} \tag{8}$$

$$\dot{\eta} = q(e + y_d, \eta),$$

where y_d is the desired output.

From input–output linearization technique [1], control input u is chosen as

$$u = \frac{1}{b(e + y_d, \eta)} \left(-a(e + y_d, \eta) + y_d^{(r)} - \sum_{i=1}^{r} c_{(i-1)} \left(\xi_i - y_d^{(i-1)} \right) \right). \tag{9}$$

such that $y(t)\backslash, \to \backslash y_d$, $t \to \infty$. The control law (9) can only be used if the system (1)–(2) is a minimum phase. Furthermore, for handling the case $b((\xi, \eta)(t_s)) = 0$ for a $t = t_s$, we used the polynomial control [7]

$$u_s(t) = \sum_{i=0}^{m-1} \frac{u^i(t_s)}{i!} (t - t_s)^i, \quad t \in [t_s - \epsilon, t_s + \epsilon], \forall \epsilon > 0. \tag{10}$$

The value $u^i(t_s)$, $i = 0, 1, \dots,$ is solution of equation systems

$$y_d^{r+k}(t_s) = a_{r+k}(\xi(t_s), u(t_s), \dot{u}(t_s), \dots, u^{(k-1)}(t_s)), \; k \geq 1, \tag{11}$$

where $y_d(t)$ is the desired output. A problem occures if system (1)–(2) is non-minimum phase. This problem is quite difficult to solve. Most of researcher restrict their research to some special nonlinear classes only. In this paper, we present the output tracking for a class non-minimum phase affine nonlinear system, where the relative degree of the system is not well defined, with relative degree being $n-1$, n is the dimension of the system. To achieve the output tracking, we will modify the steepest descent control. The modified version of steepest descent control is as follows:

$$\dot{u} = -\frac{\partial F}{\partial u} + v, \tag{12}$$

where F is a descent function which has a variable as solution of internal dynamics system. So, the design of dynamic feedback control cannot be initiated from the output causing the system to be non-minimum phase. Therefore, the output of the system will be redefined such that the system will become minimum phase with respect to a new output. An artificial input v will be determined such that the descent function becomes minimum.

3. Output tracking

We will investigate the output tracking for a non-minimum phase nonlinear system. The non-minimum phase system in the following form

$$\dot{x} = \mathbf{A}x + bu + \phi(x), \; x(t) \in \mathbf{R}^n, \; u(t) \in \mathbf{R} \tag{13}$$

$$y = x_1 \tag{14}$$

in which $\phi(x)$ is smooth vector field in $\mathbf{R}^n$, with $\phi(0) = 0$, $b = [0, \dots, 0, b_{n-1}, b_n]^T$, $b_{n-1} \neq 0$, $b_{n-1} = -b_n$, $\phi(x) = [0, \dots, 0, \phi_{n-2}(x), \phi_{n-1}(x), \phi_n(x)]^T$, where $\phi_{n-2}(x(t)) = yh_1(x_{n-1})$, $\phi_{n-1}(x) = yh_2(x_{n-1})$, $\phi_n(x) = yh_3(x_{n-1})$, with

$$\phi_{n-2}(x) + \phi_{n-1}(x) + \phi_n(x) = 0, \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

Consider the nonlinear system (13)–(14). We have

$$\begin{aligned}
\dot{x_1} &= x_2 \\
\ddot{x_1} &= x_3 \\
&\vdots \\
x_1^{(n-2)} &= x_{n-1} + x_1 h_1(x_{n-1}) \\
x_1^{(n-1)} &= \dot{x}_{n-1} + \frac{d}{dt} x_1 h_1(x_{n-1}) \\
&= \dot{x}_{n-1}\left(1 + x_1\left(\frac{dh_1(x_{n-1})}{dx_{n-1}}\right)\right) + h_1(x_{n-1})\dot{x_1} \\
&= (x_n + \phi_{n-1}(x))\left(1 + x_1\left(\frac{dh_1(x_{n-1})}{dx_{n-1}}\right)\right) \\
&\quad + (b_{n-1}u)\left(1 + x_1\left(\frac{dh_1(x_{n-1})}{dx_{n-1}}\right)\right) + h_1(x_{n-1})\dot{x_1}
\end{aligned}$$

Thus the relative degree of the system (13)–(14) is $n-1$.

The system (13)–(14), can be transformed to

$$\dot{\xi}_k = \xi_{k+1}, \qquad k = 1, \dots, n-2 \tag{15}$$

$$\dot{\xi}_{n-1} = a(\xi, \eta) + b(\xi, \eta)u \tag{16}$$

$$\begin{aligned}
\dot{\eta} &= \dot{x}_{n-1} + \dot{x}_n \\
&= \eta - x_{n-1} + yh_2(x_{n-1}) + yh_3(x_{n-1}) \\
y &= \xi_1.
\end{aligned}$$

Then the zero dynamic of the system (13)–(14) is

$$\dot{\eta} = \eta.$$

Thus the system (13)–(14) is non-minimum phase.

Our objective is to make the output system (13)–(14) track the desired output while keeping the state bounded. To make the system (13)–(14) track the desired output, we will use the dynamic feedback control. Therefore, the system output (13) will be redefined such that it becomes minimum phase. In this paper, the output that will be selected is a linear combination of the state variables. The following Lemma states that the output of the system can be chosen such that the system (13) is minimum phase.

Lemma 1. *Consider system (13). Then there exists a linear combination of the state variables $\mu = cx = [c_1\ c_2\ \ldots\ c_n]x$ such that the relative degree of the system (13) with respect to output μ is $n-1$. Furthermore the system (13) with respect to output μ is minimum phase.*

Proof. Let $\mu = cx = [c_1\ c_2\ \ldots\ c_n]x$. We will choose $c_1, c_2, \ldots, c_n$ such that the relative degree of the system (13) with respect to output μ is $n-1$. We have

$$\dot{\mu} = c\dot{x} = cAx + cbu + t\phi(x)$$

If $(cb)u = 0$, then $c_{n-1} = c_n$. Furthermore if $c\phi(x) = 0$, then $c_{n-2} = c_{n-1} = c_n$.

By $cx = [c_1\ c_2\ \ldots\ c_{n-2}\ c_{n-2}\ c_{n-2}]x$, we have

$$\ddot{\mu} = cA^2x + cAbu + cA\phi(x) = cA^2x + cA\phi(x)$$

If $cA\phi(x) = 0$, then $c_{n-3} = c_{n-2}$.

By $cx = [c_1\ c_2\ \ldots\ c_{n-3}\ c_{n-3}\ c_{n-3}\ c_{n-3}]x$, we have

$$\dddot{\mu} = cA^3x + cA^2\phi(x)$$

Next, we choose $c_2 = c_3 = c_4 = \ldots = c_n$. We have

$$\mu^{(n-2)} = cA^{(n-2)}x + cA^{(n-3)}\phi(x). \qquad (17)$$

If $c_1 \neq t_2$, then $cA^{(n-3)}\phi(x) = x_1 h(x_{n-1})$.

Thus

$$\begin{aligned}\mu^{(n-1)} &= \frac{d}{dt}cA^{(n-2)}x + \frac{d}{dt}(x_1 h(x_{n-1}))\\ &= c_1(x_n + \phi_{n-1}(x)) + c_2\phi_n(x) + \dot{x}_1 h(x_{n-1})\\ &\quad + x_1\frac{dh(x_{n-1})}{dx_{n-1}}(x_n + \phi_{n-1}(x))\\ &\quad + b_{n-1}u\left(c_1 - c_2 + x_1\frac{dh(x_{n-1})}{dx_{n-1}}\right).\end{aligned}$$ □

Therefore there exists the linear combination of the state variables $\mu = cx = [c_1\ c_2\ \ldots\ c_n]x$, with $c_2 = c_3 = \ldots = c_n$, $c_1 \neq c_2$ such that the relative degree of the system (13) with respect to output μ is $n-1$.

Furthermore the internal dynamic (13) in respect to output μ is

$$\begin{aligned}\dot{\eta} &= \dot{x}_{n-1} + \dot{x}_n = x_n + \phi_{n-1}(x) + \phi_n(x)\\ &= x_n - \phi_{n-2}(x)\end{aligned} \qquad (18)$$

From Eq. (17), we have

$$\begin{aligned}\mu^{(n-2)} &= cA^{(n-2)}x + cA^{(n-3)}\phi(x)\\ &= c_1x_{n-1} + c_2x_n + t_1\phi_{n-2}(x) + c_2(\phi_{n-1}(x) + \phi_n(x)).\end{aligned}$$

We choose $c_1 = c_2 - 1$, $c_2 \geq 2$. Then

$$\begin{aligned}\mu^{(n-2)} &= c_1x_{n-1} + c_2x_n - \phi_{n-2}(x)\\ &= c_1x_{n-1} + c_1x_n + x_n - \phi_{n-2}(x).\end{aligned} \qquad (19)$$

Thus, the internal dynamic is

$$\begin{aligned}\dot{\eta} &= x_n + \mu^{(n-2)} - c_1(x_{n-1} + x_n) - x_n\\ &= \mu^{(n-2)} - c_1\eta.\end{aligned} \qquad (20)$$

Therefore, the zero dynamic (13) in respect to output μ is

$$\dot{\eta} = -c_1\eta. \qquad (21)$$

Hence, there exists the linear combination of the state variables $\mu = cx = [c_1\ c_2\ \ldots\ c_n]x$, with $c_2 = c_3 = \ldots = t_n$, $c_1 = c_2 - 1$, $c_2 \geq 2$ such that the system (13) in respect to output μ is minimum phase.

Let $\mu_d = (c_1\ c_2\ \ldots\ c_2)x_d$, with $x_d = (x_{1d}\ x_{2d}\ \ldots\ x_{nd})^T$ is the desired output of the output which has been selected. Furthermore will be set based on the desired output of the original system.

Assumption 1. Substituion $x_i = x_{id}$, $i = 1, 2, \ldots, n-2$.

Based on assumption 1 we have $x_{2d}, x_{3d}, \ldots, x_{(n-1)d}$, respectively. Then $\dot{x}_n = f(x_1, x_{n-1}, x_n)$ can be solved by substituting $x_{n-1} = x_{(n-1)d}$. Thus $x_n = x_{nd}$. Furthermore definition error $e = \mu - \mu_d$, with $\mu_d = t_1 x_{1d} + t_2 x_{2d} + \cdots + t_2 x_{nd}$.

We design a control law u through properties of the solution of higher order ordinary differential equation. Consider a differential equation

$$a_r e^{(r)}(t) + a_{r-1} e^{(r-1)}(t) + \cdots + a_1 \dot{e}(t) + a_0 e(t) = 0, \tag{22}$$

where r is the relative degree of the system. If a polynomial

$$p(s) = a_r s^r + a_{r-1} s^{r-1} + \cdots + a_1 s + a_0 \tag{23}$$

is Hurwitz, then solution of differential Eq. (22) tends to zero if $t \to \infty$. In this case for the purpose of designing the control law required an explicit relationship between input and output. For that, we define a descent function as follows:

$$\begin{aligned} F(\mu, \mu_d, \dot{\mu}, \dot{\mu}_d, \ldots, \mu^{(n-1)}(t), \mu_d^{(n-1)}(t)) &= \left(\sum_{j=0}^{n-1} a_j (\mu - \mu_d)^{(j)} \right)^2 \\ &= \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right)^2. \end{aligned} \tag{24}$$

By "Trajectory Following Method" [12], the control u is determined from the differential equation

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_{n-1} \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) \frac{\partial e^{(n-1)}}{\partial u} \tag{25}$$

The control law in Eq. (25) is called the steepest descent control.

From Eq. (16), then

$$\begin{aligned} \mu^{(n-1)} &= t_1 \dot{x}_{n-1} + (t_1 + 1)\dot{x}_n - \frac{d}{dt}(\phi_{n-2}(x)) \\ &= t_1 (x_n + \phi_{n-1}(x)) + (t_1 + 1)\phi_n(x) - \dot{x}_1 h_1(x_{n-1}) - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} (x_n + \phi_{n-1}(x)) \\ &\quad + u\left(-b_{n-1} - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} b_{n-1} \right) \end{aligned} \tag{26}$$

From Eq. (26), then

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_{n-1} \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) \left(-b_{n-1} - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} b_{n-1} \right) \tag{27}$$

Calculate the time derivative of the descent function (24) along the trajectory of the extended system

$$\dot{x} = Ax + bu + \phi(x), \tag{28}$$

$$\dot{u} = -2a_{n-1} \left(\sum_{j=0}^{n-1} a_j (e)^{(j)} \right) \times \left(-b_{n-1} - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} b_{n-1} \right). \tag{29}$$

Then we have

$$\begin{aligned} \dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) &= \frac{\partial F}{\partial e}\dot{e} + \frac{\partial F}{\partial \dot{e}}\ddot{e} + \cdots + \frac{\partial F}{\partial e^{(n-1)}} e^{(n)} \\ &= 2\left(\sum_{j=0}^{n-1} a_j (\mu - \mu_d)^{(j)} \right) \left[\sum_{j=0}^{n-3} a_j \left(tA^{(j+1)} x - \mu_d^{(j+1)} \right) + a_{n-3}(-\phi_{n-2}(x)) + a_{n-2} \left(\frac{d}{dt} \left(tA^{(n-2)} x - \phi_{n-2}(x) \right) - \mu_d^{(n-1)} \right) \right] \\ &\quad + 2\left(\sum_{j=0}^{n-1} a_j (\mu - \mu_d)^{(j)} \right) \times \left[a_{n-1} \left(\frac{d}{dt} (t_1 (x_n + \phi_{n-1}(x) + (t_1 + 1)\phi_n(x)) \right. \right. \\ &\quad - \frac{d}{dt} \left(\dot{x}_1 h_1(x_{n-1}) - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} (x_n + \phi_{n-1}(x) \right) - \mu_d^{(n)} + \frac{d}{dt} \left(-b_{n-1} - x_1 \frac{dh(x_{n-1})}{dx_{n-1}} b_{n-1} \right) u \\ &\quad \left. \left. + \left(-b_{n-1} - x_1 \left(\frac{dh(x_{n-1})}{dx_{n-1}} b_{n-1} \right) \right) \dot{u} \right) \right] \end{aligned} \tag{30}$$

From Eq. (29), then according to (30) the value of the time derivative of the descent function (24) along the trajectory of (28)–(29) cannot be guaranteed to be less than zero $t \geq 0$.

Consider the extended system (28)–(29) and the time derivative of the descent function (30). We do not have a variable which will be used to push the time derivative of descent function (30) less than zero. Now we modify the steepest descent (25) by adding an artificial input v. Then the extended system (28)–(29) becomes

$$\dot{x} = Ax + bu + \phi(x), \tag{31}$$

$$\dot{u} = -\frac{\partial F}{\partial u} + v. \tag{32}$$

The control law in Eq. (32) is called as modified steepest descent control.

By the same way, let us calculate the time derivative of the descent function (24) along the trajectory system (31)–(32) yielding

$$\begin{aligned}\dot{F}(e,\dot{e},\dots,e^{(n-1)}) &= \frac{\partial F}{\partial e}\dot{e} + \frac{\partial F}{\partial \dot{e}}\ddot{e} + \cdots + \frac{\partial F}{\partial e^{(n-1)}}e^{(n)} = 2\left(\sum_{j=0}^{n-1} a_j(\mu-\mu_d)^{(j)}\right)\left[\sum_{j=0}^{n-3} a_j\left(tA^{(j+1)}x - \mu_d^{(j+1)}\right)\right. \\ &\left.+ a_{n-3}(-\phi_{n-2}(x)) + a_{n-2}\left(\frac{d}{dt}\left(tA^{(n-2)}x - \phi_{n-2}(x)\right) - \mu_d^{(n-1)}\right)\right] + 2\left(\sum_{j=0}^{n-1} a_j(\mu-\mu_d)^{(j)}\right) \\ &\times\left[a_{n-1}\left(\frac{d}{dt}(t_1(x_n + \phi_{n-1}(x) + (t_1+1)\phi_n(x)) - \frac{d}{dt}\left(\dot{x}_1 h_1(x_{n-1}) - x_1\frac{dh_1(x_{n-1})}{dx_{n-1}}(x_n + \phi_{n-1}(x)\right)\right.\right. \\ &\left.\left. - \mu_d^{(n)} + \frac{d}{dt}\left(-b_{n-1} - x_1\frac{dh(x_{n-1})}{dx_{n-1}}b_{n-1}\right)u\right] - \left(2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e)^{(j)}\right)\left(-b_{n-1} - x_1\frac{dh_1(x_{n-1})}{dx_{n-1}}b_{n-1}\right)\right)^2 \\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e)^{(j)}\right)\left(-b_{n-1} - x_1\frac{dh_1(x_{n-1})}{dx_{n-1}}b_{n-1}\right)v\end{aligned} \tag{33}$$

Consider Eq. (33). We will choose the artificial input v such that $\dot{F}(e_1,\dot{e_1},\dots,e_1^{(n-1)})$ be less than zero. If we take

$$\begin{aligned} v = &\left(\frac{1}{\frac{\partial F}{\partial u}}\right)\left(-k(e,\dot{e},\dots,e^{(n-1)})\right. \\ &\left.-\sqrt{k\left(e,\dots,e^{(n-1)}\right)^2 + \left(\frac{\partial F}{\partial u}\right)^2}\right), \end{aligned}\tag{34}$$

where

$$\begin{aligned}k(e,\dot{e},\dots,e^{(n-1)}) &= 2\left(\sum_{j=0}^{n-1} a_j(\mu-\mu_d)^{(j)}\right)\left[\sum_{j=0}^{n-3} a_j\left(tA^{(j+1)}x - \mu_d^{(j+1)}\right)\right. \\ &\left.+ a_{n-3}(-\phi_{n-2}(x)) + a_{n-2}\left(\frac{d}{dt}\left(tA^{(n-2)}x - \phi_{n-2}(x)\right) - \mu_d^{(n-1)}\right)\right] + 2\left(\sum_{j=0}^{n-1} a_j(\mu-\mu_d)^{(j)}\right) \\ &\times\left[a_{n-1}\left(\frac{d}{dt}(t_1(x_n + \phi_{n-1}(x) + (t_1+1)\phi_n(x)) - \frac{d}{dt}\left(\dot{x}_1 h_1(x_{n-1}) - x_1\frac{dh_1(x_{n-1})}{dx_{n-1}}(x_n + \phi_{n-1}(x)\right) - \mu_d^{(n)}\right.\right. \\ &\left.\left.+ \frac{d}{dt}\left(-b_{n-1} - x_1\frac{dh(x_{n-1})}{dx_{n-1}}b_{n-1}\right)u\right)\right]\end{aligned} \tag{35}$$

Then

$$\dot{F}(e_1,\dot{e_1},\dots,e_1^{(r)}) = -\left(\frac{\partial F}{\partial u}\right)^2 - \sqrt{k\left(e_1,\dots,e_1^{(r)}\right)^2 + \left(\frac{\partial F}{\partial u}\right)^2} \tag{36}$$

Assumption 2. We choose $y_d = x_{1d}$ such that $-b_{n-1}\left(1 + x_{1d}\frac{dh(x_{n-1})}{dx_{n-1}}\right) \neq 0$, where y_d is the desired output.

Theorem 1. *Consider systems (13), with the output $\mu = cx = [c_1\ c_2\ \dots\ c_n]x$, where $c_2 = c_3 = \dots = c_n$, $c_1 = c_2 - 1$, $c_2 \geq 2$. Choose constants a_i such that the polynomial*

$$p(s) = a_0 + a_1 + \cdots + a_{n-2}s^{n-2} + a_{n-1}s^{n-1} \tag{37}$$

is Hurwitz. Then, by using the modified steepest descent control

$$\dot{u} = -2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e)^{(j)}\right)\left(-b_{n-1} - x_1 \frac{dh_1(x_{n-1})}{dx_{n-1}} b_{n-1}\right) + v, \tag{38}$$

with v as in Eq. (34), error e tend to zero if time t goes to infinity, where $e = \mu - \mu_d$. Furthermore the output of the original system $y = x_1$ track to the desired output $y_d(t)$.

Proof. From Eq. (36), we have $\dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) < 0$, if $\sum_{j=0}^{n-1} a_j(e)^{(j)} \neq 0$. Let $\sum_{j=0}^{n-1} a_j(e)^{(j)} = 0$. From Eq. (36), $\dot{F}(e, \dot{e}, \ldots, e^{(n-1)}) = 0$.

Thus, the descent function (24) becomes minimum. The minimum value of descent function (24) is zero.

Therefore, if $F(e, \dot{e}, \ldots, e^{(n-1)}(t))$ is zero, then $\sum_{j=0}^{n-1} a_j(e)^{(j)} = 0$.

Furthermore by assumption 2, if $\frac{\partial F}{\partial u} = 0$, then $\sum_{j=0}^{n-1} a_j(e)^{(j)} = 0$. Thus, we choose a_j, $j = 0, \ldots, n-1$ such that the polynomial $p(s) = a_0 + a_1 s + \cdots + a_{r-1}s^{n-2} + s^{n-1}$ is Hurwitz, then error e goes to zero as $t \to \infty$. Thus μ tend to μ_d if time t goes to infinity. Hence the output of the original system $y = x_1$ tracks to the desired output $y_d(t)$. □

4. Example

Consider the nonlinear system (SISO)

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3 + x_1 x_3 \\ \dot{x_3} &= x_4 - u + x_1 x_3 \\ \dot{x_4} &= u - 2x_1 x_3 \end{aligned} \tag{39}$$

$$y = x_1. \tag{40}$$

The nonlinear system (39)–(40) has relative degree 3 at any point x_0 (relative degree of the system is not well defined). Because the stability of zero dynamic is unstable, the nonlinear system (39)–(40) is the non-minimum phase. Now, redefining output $z_1 = x_1 + 2x_2 + 2x_3 + 2x_4$.

By considering the new output, the relative of the system (39) is 3 at any point x_0 ((relative degree of the system is not well defined). The system (39) in normal form with respect to output z_1

$$\begin{aligned} \dot{z_1} &= z_2 \\ \dot{z_2} &= z_3 \\ \dot{z_3} &= a(z) + b(z)u \\ \dot{\eta} &= -\eta + z_3, \end{aligned} \tag{41}$$

with $a(z) = x_4 - x_1^2 x_3 - 3x_1x_3 - x_2x_3 - x_1x_4$, $b(z) = 1 + x_1$. Thus the system (39) is the minimum phase with respect to the new output.

Then according to (9) and (10), the static control law is

$$u(t) = \begin{cases} \frac{1}{b(z)}(-a(z) + z_{1d}^{(3)}) - \sum_{i=1}^{3} c_{(i-1)}\left(z_i - z_{1d}^{(i-1)}\right) & ; t \in [0, t_s - \epsilon] \cup [t_s + \epsilon, \infty] \\ \sum_{i=0}^{2} \frac{u^{(i)}(t_s)}{i!}(t - t_s)^i & ; t \in [t_s - \epsilon, t_s + \epsilon], \end{cases} \tag{42}$$

with $\epsilon = 0.001$ and according to (38), the modified steepest descents control with respect to z_1 is

$$\dot{u} = -2(1 + x_1)a_3\left(a_0(z_1 - z_{1d}) + a_1(\dot{z_1} - \dot{z_{1d}}) + a_2(\ddot{z_1} - \ddot{z_{1d}}) + a_3\left(z_1^{(3)} - z_{1d}^{(3)}\right)\right) + v, \tag{43}$$

where v as in Eq. (34).

(i) Let $y_d(t) = x_{1d}(t) = 0.5sin(t)$. Next, we choose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$, then $y(t)$ tracks the desired output $y_d(t)$. By replacing x_1 with $x_{1d}(t) = 0.5sin(t)$, then $x_{2d} = 0.5cos(t)$. By replacing x_2 with $x_{2d}(t)$, then $x_{3d} = -\frac{0.5sin(t)}{1+0.5sin(t)}$. By replacing x_3 with $x_{3d}(t)$, we have a differential equation $\dot{x_4} - x_4 = \frac{-0.5cos(t)}{(1+0.5sin(t))^2} + \frac{0.25sin^2(t)}{1+0.5sin(t)}$.

Thus $x_{4d} = 1/2(-0.5cos(t) - 0.5sin(t) + \frac{sin(t)}{1+0.5sin(t)})$. Now, $z_{1d} = 0.5cos(t)$.

Simulation results for the static control law (42) are shown in Fig. 1(a) and (b) for constants $c_0 = 1$, $c_1 = 2$, $c_2 = 7$. Initial value $x_1(0) = 0$, $x_2(0) = 0.5$, $x_3(0) = 0$, $x_4(0) = -0.5$.

Simulation results for the modified steepest descent control (43) are shown in Fig. 2(a) and (b) for constants $a_0 = 15$, $a_1 = 23$, $a_2 = 9$, $a_3 = 1$. Initial value $x_1(0) = 0$, $x_2(0) = 0.5$, $x_3(0) = 0$, $x_4(0) = -0.5$, $u(0) = 0.2$:

In Figs. 1(a) and 2(a), the output which has been selected such that the system become minimum phase track the desired output $z_{1d} = 0.5cos(t)$.

In Figs. 1(b) and 2(b), the output of the original system track the desired output $y_d = 0.5sin(t)$.

In Fig. 2(c), the response curve of control input.

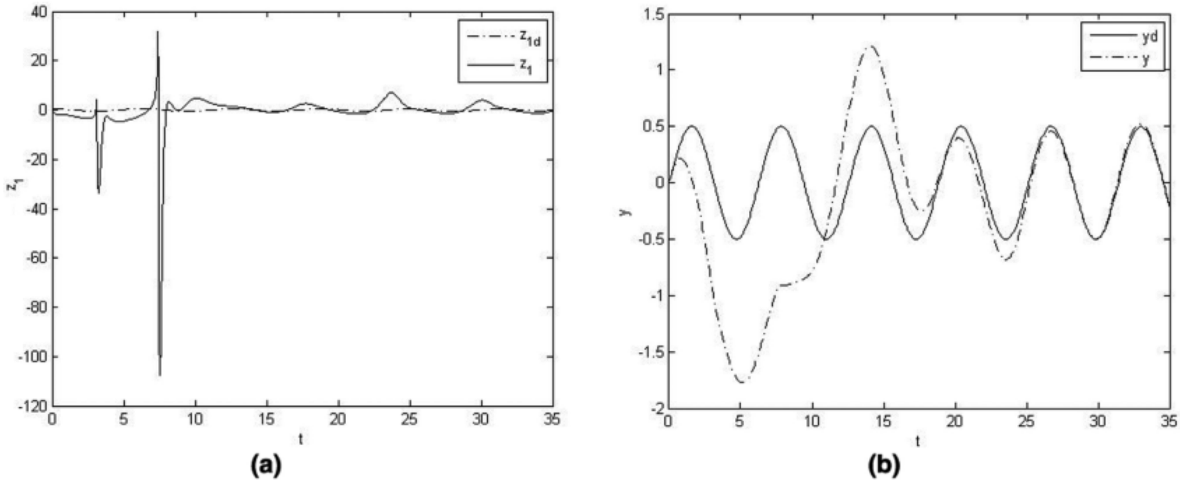

Fig. 1. Static Control law. Initial value :$x_1(0) = 0$, $x_2(0) = 0.5$, $x_3(0) = 0$, $x_4(0) = -0.5$; Constants $c_0 = 1$, $c_1 = 2$, $c_2 = 7$, $z_{1d} = 0.5cos(t)$, $y_d = 0.5sin(t)$.

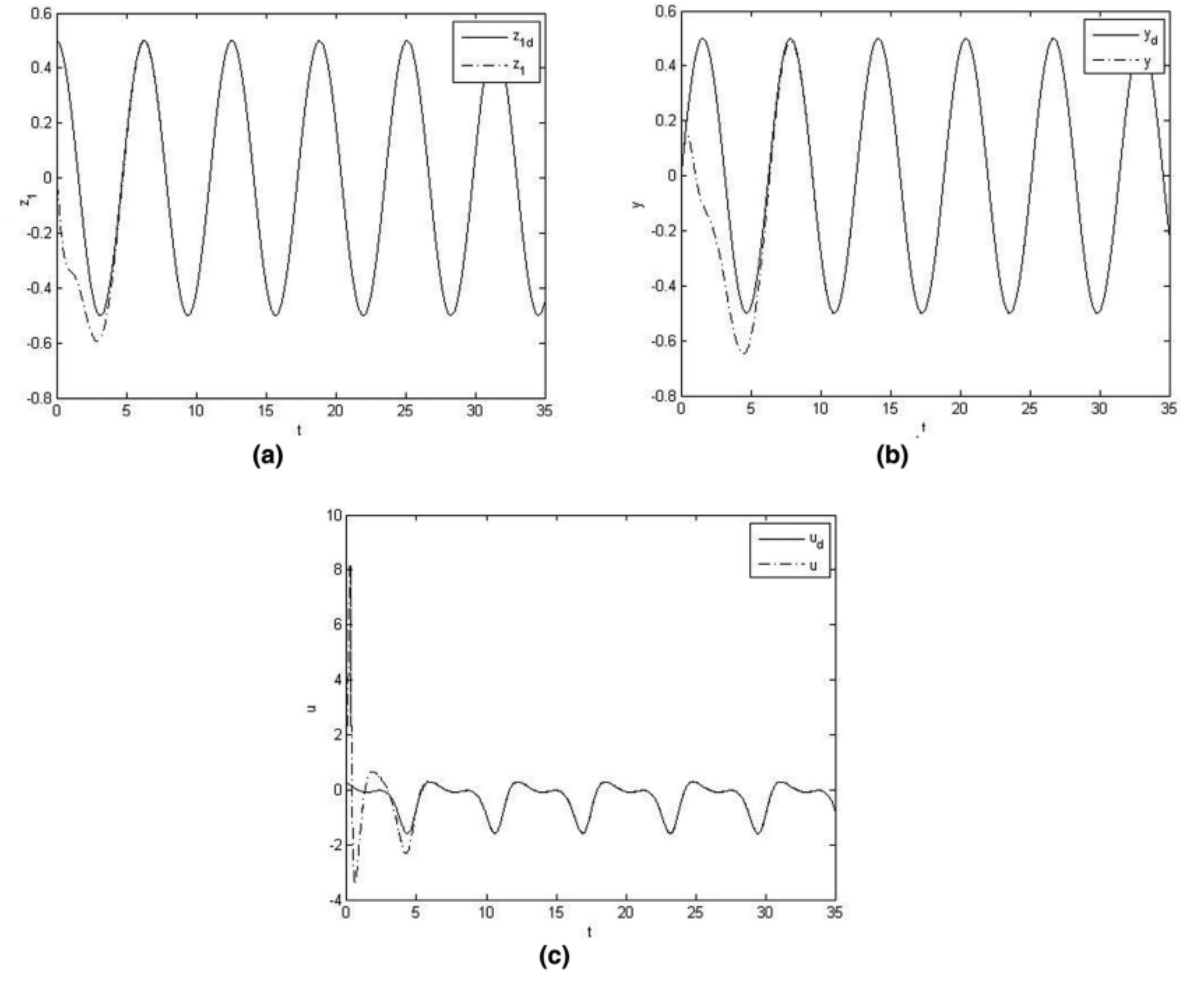

Fig. 2. The simulation results of the modified Steepest Descent Control.

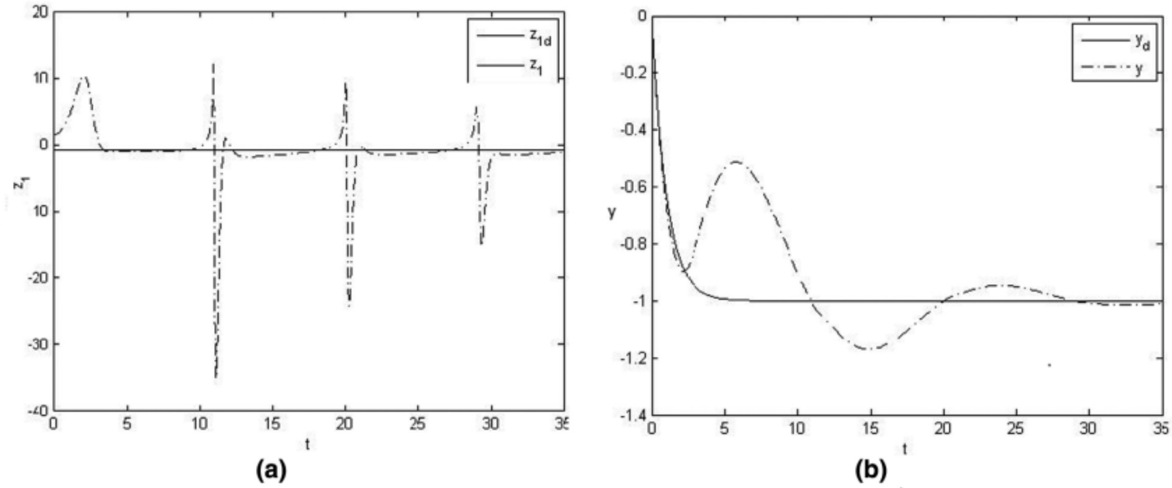

Fig. 3. Static control law. Initial value :$x_1(0) = 0$, $x_2(0) = -1$, $x_3(0) = 1$, $x_4(0) = -0.5$; Constants $c_0 = 1$, $c_1 = 2$, $c_2 = 6$.

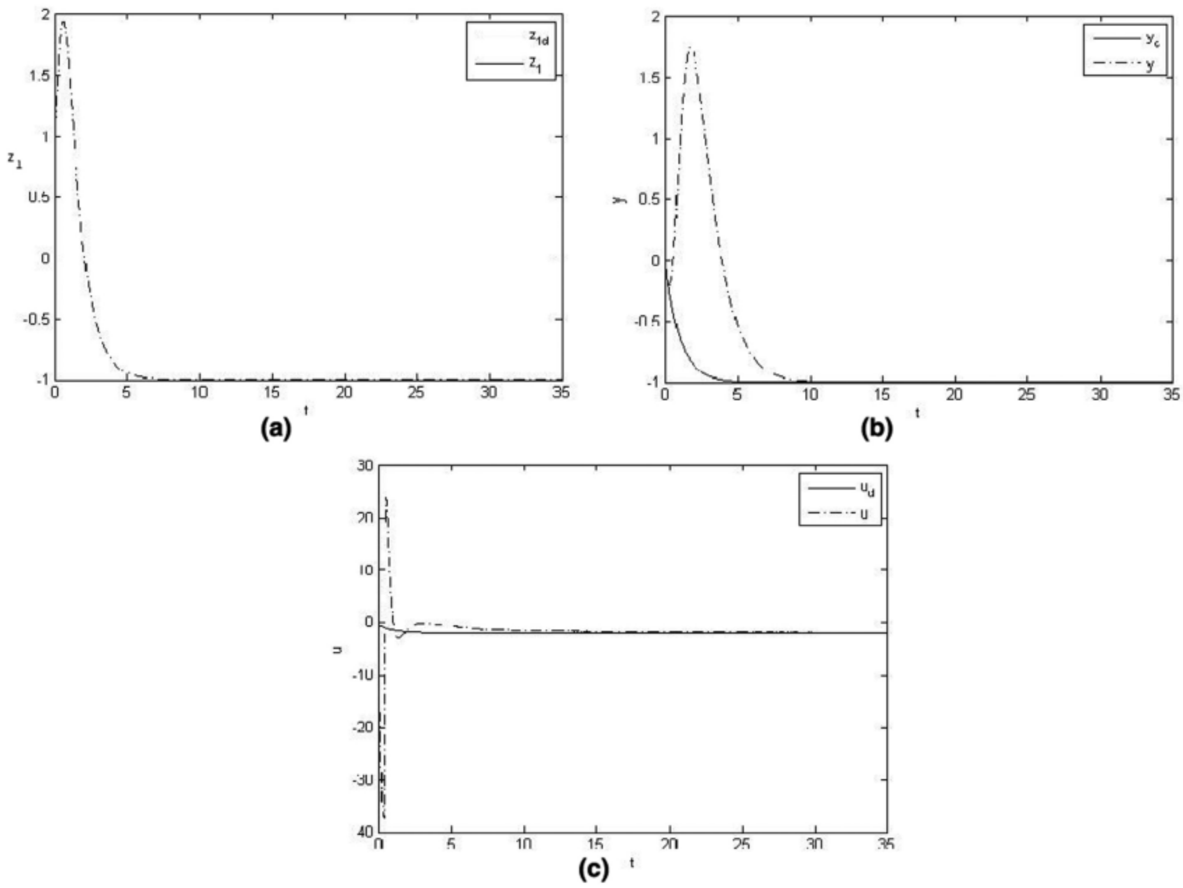

Fig. 4. Modified steepest descent control (a) Output tracking z_1 to $z_{1d} = -1$.(b) Output tracking y to $y_d = e^{-t} - 1$. (c) the input control u.

(ii) Let $y_d(t) = e^{-t} - 1$. Next, we choose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$, then $y(t)$ tracks to the desired output $y_d(t)$. By replacing x_1 with $x_{1d}(t) = e^{-t} - 1$, then $x_{2d} = -e^{-t}$. By replacing x_2 with $x_{2d}(t)$, then $x_{3d} = 1$. By replacing x_3 with $x_{3d}(t)$, we have a differential equation $\dot{x_4} - x_4 = 1 - e^{-t}$.
Thus $x_{4d} = 0.5e^{-t} - 1$. Now, $z_{1d} = -1$.
Simulation results for the static control law (42) are shown in Fig. 3(a) and (b) for constants $c_0 = 1$, $c_1 = 2$, $c_2 = 7$. Initial value $x_1(0) = 0$, $x_2(0) = -1$, $x_3(0) = 1$, $x_4(0) = -0.5$.
Simulation results for the modified steepest descent control (43) are shown in Fig. 4(a) and (b) for constants $a_0 = 15$, $a_1 = 23$, $a_2 = 9$, $a_3 = 1$. Initial value $x_1(0) = 0$, $x_2(0) = -1$, $x_3(0) = 1$, $x_4(0) = -0.5$, $u(0) = -0.5$:
In Figs. 3(a) and 4(a), the output which has been selected such that the system become minimum phase track the desired output $z_{1d} = -1$.
In Figs. 3(b) and 4(b), the output of the original system track the desired output $y_d = e^{-t} - 1$.
In Fig. 4(c), the response curve of control input.

5. Conclusions

In this paper, we have investigated the output tracking for a class of nonlinear non-minimum phase system (13)–(14). The dynamic feedback control has been designed for the output tracking. The design of the dynamic control is based on the modification of the steepest descent control. To perform the design of the modified steepest descent control, the systems (13) are required to be minimum phase with respect to a new output, where the new output is the linear combination of the state variables. Furthermore, the new desired output will be set based on the desired output of the original system. By applying the modified steepest descent control, the system output tracks the desired output.

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References

[1] A. Isidori, Nonlinear control systems: An introduction, third edition, Springer-Verlag, Berlin, Heidelberg, 1995.
[2] H.K. Khalil, Nonlinear systems, third edition, Prentice Hall, New Jersey, 2002.
[3] D. Chen, B. Paden, Stable inversion of nonlinear non-minimum phase systems, Int.J.Control 64 (1) (1996) 45–54.
[4] D. Chen, Iterative solution to stable inversion of nonlinear non-minimum phase systems, Proceedings of the American Control Conference (1993) 2960–2964.
[5] D. Li, Output tracking of nonlinear nonminimum phase systems : an engineering solution, Proceedings of the 44th IEEE Conference on Decision and control, and the European Control Conference (2005) 3462–3467.
[6] S. Baev, Y. Shtessel, I. Shkolnikov, Hosm driven output tracking in the non-minimum-phase causal nonlinear systems, Proceedings of the 46th IEEE Conference on Decision and Control (2007) 3715–3720.
[7] J. Naiborhu, Firman, K. Mu'tamar, Particle swarm optimization in the exact linearization technic for output tracking of non-minimum phase nonlinear systems, Appl. Math. Sci. 7 (109) (2013) 5427–5442.
[8] R. Marino, P. Tomei, A class of globally output feedback stabilizable nonlinear non-minimum phase systems, IEEE Trans. Autom. Control 50 (12) (2005) 2097–2101.
[9] Z.Y. Zili Li Zengqiang Chen, The stability analysis and control of nonminimum phase nonlinear systems, Int. J. Nonlinear Sci. 3 (2) (2007) 103–110.
[10] F.C.N. Wang, W. Xu, Adaptive global output feedback stabilisation of some non-minimum phase nonlinear uncertain systems, IET Control Theory Appl. 2 (2) (2008) 117–125.
[11] J. Naiborhu, Trajectory following method on output regulation of affine nonlinear control systems with relative degree not well defined, ITB J. Sci. 43 A (2) (2011) 73–86.
[12] T.L. Vincent, W.J. Grantham, Nonlinear and optimal control systems, John Wiley & Sons, Inc., New York, 1997.

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