Output Tracking of Some Class Non-Minimum Phase Nonlinear Systems Via Output Redefinition

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Output Tracking of Some Class Non-Minimum Phase Nonlinear Systems Via Output Redefinition

Firman, Janson Naiborhu

Industrial & Financial Mathematics Group, ITB

Abstract. In this paper, we present the output tracking for a class non-minimum phase nonlinear. To achive the output tracking, we will apply the modified steepest descent control. To apply the modified steepest descent control, the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

INTRODUCTION

In the output tracking theory, the input output linearization is one of the most available methods [1]. Output tracking problems for nonlinear non-minimum phase systems is a rather difficult issue in control theory. Most of researcher restrict their research to some special nonlinear classes only. The stable inversion proposed in [2], [3] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on the imaginary axis. In [4], control design procedure for the output tracking was proposed. The design procedure consists of two steps. In the first step, the standard input output linearization is applied. In the second step, we group an output with the internal dynamics as one subsystems, which is usually nonlinear, and the rest of the output as the other subsystem that is linear, the nonlinear subsystems is linearized about its equilibrium. In [5], the asymptotic output tracking which is a class of causal nonminimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. In [6], J. Naiborhu *et.al* have developed a method to design the input control to track the output of a non-minimum phase nonlinear systems asymptotically. The design of the input control is based on the exact linearization. To perform an exact linearization, the output should be selected such that its relative degree is equal to the dimension of the system. Results on stabilization of non-minimum phase system in the output feedback form have been presented in In [7], [8], [9]. The main idea in [7], [8], [9] is output reconstruction such that becomes minimum phase with respect to a new output.
In this paper, we will modify the steepest descent control for output tracking of a class non-minimum phase nonlinear uncertain systems, with relative degree being $n-1$, n is the dimension of the system. The modification is the addition of an input artificial of the steepest descent control. The design of descent control can not be initiated from the output causing the system to be non-minimum phase. In this paper to solve the problem, we transform the system into a normal form which is minimum phase with respect to a virtual output, which is a linear combination of state variables.

Problem Statement

Consider affine nonlinear system

$$\dot{x} = \mathbf{A}x + bu + \phi(y), \; x(t) \in \mathbf{R}^n, \; u(t) \in \mathbf{R} \tag{1}$$

$$y = x_1 \tag{2}$$

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in which $\phi(y)$ is smooth vector field in $\mathbf{R}^n$, with $\phi(0) = 0$, $b = [0, \ldots, 0, b_{n-1}, b_n]^T$, $b_{n-1} \neq 0$, $b_{n-1} = -b_n$,

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{pmatrix}$$

The relative degree of the system (1)-(2) is $n-1$. The system (1)-(2), can be transformed to

$$\begin{aligned} \dot{\omega}_k &= \omega_{k+1}, \quad k = 1, \ldots, n-2 && (3) \\ \dot{\omega}_{n-1} &= a(\omega\ , \eta) + b(\omega\ , \eta)u && (4) \\ \dot{\eta} &= \eta - x_1 + \phi_1(x_1) + \cdots + \phi_n(x_1) && (5) \\ y &= \omega_1. \end{aligned}$$

Then the zero dynamic of the system (1)-(2) is

$$\dot{\eta} = \eta.$$

Thus the system (1)-(2) is non-minimum phase.

Our objective is to make the output system (1)-(2) track the desired output. To make the system (1)-(2) track the desired output, we will use the dynamic feedback control. The design of the dynamic control is based on the modification of the steepest descent control. By "Trajectory Following Method [10], the steepest descent control is determined from the differential equation $\dot{u} = -\frac{\partial F}{\partial u}$, where F is a descent function which has a variable as solution of internal dynamics system. So, to modify the steepest descent control can not be initiated from the output causing the system to be non-minimum phase. Therefore, the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

Output Tracking

We consider system (1). Consider now a new output $\mu = \alpha x_1 + x_2 + \cdots + x_n$. The relative degree of system (1) with respect to μ is $n-1$. The system (1) with respect to μ, can be transformed to

$$\begin{aligned} \dot{z}_k &= z_{k+1}, \quad k = 1, \ldots, n-2 && (6) \\ \dot{z}_{n-1} &= a(z\ , \eta) + b(z\ , \eta)u && (7) \\ \dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \ldots + \dot{x}_n \\ &= \eta - x_1 + \phi_1(x_1) + \ldots + \phi_n(x_1) \\ \mu &= z_1. \end{aligned}$$

Furthermore

$$\eta\dot{\eta} = \eta(\eta - x_1 + \phi_1(y) + \cdots + \phi_n(y)) \quad (8)$$

Case 1 : if $\phi_1(y) + \cdots + \phi_n(y) = 0$.
Then

$$\begin{aligned} \eta\dot{\eta} &= \eta(\eta - x_1) = \eta^2 - \eta x_1 \\ &= \eta^2 - \eta\left(\frac{z_1 - \eta}{\alpha - 1}\right) \end{aligned} \quad (9)$$

Then if $z_1 = 0$ and $0 < \alpha < 1$, then

$$\eta\dot{\eta} = \frac{\alpha\eta^2}{\alpha - 1} < 0. \quad (10)$$

Therefore, the zero dynamic (1) with respect to output μ is asymptotic stable. Thus the system (1) with respect to output is minimum phase.

Case 2 : if $\phi_1(y) + \cdots + \phi_n(y) = h(y) \neq 0$.

Then

$$\eta\dot{\eta} = \eta(\eta - x_1 + h(y)) = \eta(\eta - \left(\frac{z_1 - \eta}{\alpha - 1}\right) + h\left(\frac{z_1 - \eta}{\alpha - 1}\right)$$