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Output Tracking of Some Class Non-minimum Phase Nonlinear Uncertain Systems

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Abstract: In this paper, we present the output tracking for a class of non-minimum phase nonlinear uncertain systems. To achieve the output tracking, we will apply the modified steepest descent control. To apply the modified steepest descent control, the output of the system will be redefined so that the system will become minimum phase with respect to a new output.

Keywords: *relative degree of system; minimum phase system; non-minimum phase system; modified steepest descent control.*

Mathematics Subject Classification (2010): 93C10, 93D21.

1 Introduction

In the output tracking theory, the input-output linearization is one of the most available methods [1]. Output tracking problems for nonlinear non-minimum phase systems are a rather difficult issue in control theory. Most of researchers restrict their research to some special nonlinear classes only. The stable inversion proposed in [2], [3] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on the imaginary axis. In [4], control design procedure for the output tracking was proposed. The design procedure consists of two steps. At the first step, the

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standard input-output linearization is applied. At the second step, we group an output with the internal dynamics as one subsystem, which is usually nonlinear, and the rest of the output as the other subsystem that is linear, the nonlinear subsystem is linearized about its equilibrium. In [5], the asymptotic output tracking which is a class of causal non-minimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. Results on stabilization of non-minimum phase system in the output feedback form have been presented in [6], [7], [8]. The main idea in [6], [7], [8] is output reconstruction such that the system becomes minimum phase with respect to a new output. Results on output tracking of some class non-minimum phase nonlinear system have been presented in [9], [10]. In [9], the design of the input control is based on the exact linearization.

In this paper, we will modify the steepest descent control for output tracking of a class of non-minimum phase nonlinear uncertain systems, with relative degree being $n-1$, n is the dimension of the system. The modification is the addition of an artificial input of the steepest descent control. The design of descent control can not be initiated from the output causing the system to be non-minimum phase. In this paper, to solve the problem, we transform the system into a normal form which is minimum phase with respect to a virtual output, which is a linear combination of state variables.

2 Problem Statement

Consider nonlinear uncertain system

$$\dot{x} = \mathbf{A}x + \phi(y) + \theta\psi(y) + bu, \; x(t) \in \mathbf{R}^n, \; u(t) \in \mathbf{R}, \tag{1}$$

$$y = x_1, \tag{2}$$

in which $\phi(x)$ is smooth vector field in $\mathbf{R}^n$, with $\phi(0) = 0$, $\phi(y) = [\phi_1(y), \phi_2(y), \ldots, \phi_n(y)]^T$, $\psi(0) = 0$, $\theta\psi(y) = [\theta_1(t)\psi_1(y), \theta_2(t)\psi_2(y), \ldots, \theta_n(t)\psi_n(y)]$, $b = [0, \ldots, 0, b_{n-1}, b_n]^T$,

$$b_{n-1} \neq 0, \; b_{n-1} = -b_n \text{ and } \mathbf{A} = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ 0 & 0 & \ldots & 0 \end{pmatrix}.$$

The relative degree of the system (1)-(2) is $n-1$.

The system (1)-(2) can be transformed to

$$\begin{aligned}
\dot{z}_1 &= z_2 + \theta_1(t)\psi(x_1), && (3)\\
\dot{z}_k &= z_{k+1} + \varphi_{k-1}(t, x_1, \ldots, x_{k-1}), \quad k = 2, \ldots, n-2, && (4)\\
\dot{z}_{n-1} &= a(z\ ,\ \eta) \ + \ b(z\ ,\ \eta)u + \varphi(t, x_1, \ldots, x_{n-2}), && (5)\\
\dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \ldots + \dot{x}_n \\
&= \eta - z_1 + \phi_1(y) + \ldots + \phi_n(y) + \theta_1(t)\psi_1(y) + \ldots + \theta_n(t)\psi_n(y), \\
y &= z_1,
\end{aligned}$$

with the internal dynamics

$$\dot{\eta} = \eta - z_1 + \phi_1(y) + \ldots + \phi_n(y) + \theta_1(t)\psi_1(y) + \cdots + \theta_n(t)\psi_n(y). \tag{6}$$

Then the zero dynamics of the system (1)-(2) is

$$\dot{\eta} = \eta.$$

Thus the system (1)-(2) is non-minimum phase.

Our objective is to make the output system (2) track the desired output. To make the system (1)-(2) track the desired output, we will use the dynamic feedback control. The design of the dynamic control is based on the modification of the steepest descent control. By "Trajectory Following Method" [11], the steepest descent control is determined from the differential equation $\dot{u} = -\frac{\partial F}{\partial u}$, where F is a descent function which has a variable as the solution of internal dynamics system. So, the modification of the steepest descent control can not be initiated from the output causing the system to be non-minimum phase. Therefore, the output of the system will be redefined so that the system will become minimum phase with respect to a new output.

3 Main Results

We consider system (1). Consider now a new output $\mu = t_1 x$, with $t_1 = (\alpha \;\; 1 \;\; 1 \;\; \dots \;\; 1)$. The relative degree of system (1) with respect to μ is $n-1$. The system (1) with respect to μ, can be transformed to

$$\dot{z}_1 = z_2 + c\theta(t)\psi(x_1), \tag{7}$$

$$\dot{z}_k = z_{k+1} + \omega_{i-1}(t, x_1, \dots, x_{i-1}), \qquad k = 2, \dots, n-2, \tag{8}$$

$$\dot{z}_{n-1} = a(z\ ,\ \eta) \ + \ b(z\ ,\ \eta)u + \omega(t, x_1, \dots, x_{n-2}), \tag{9}$$

$$\begin{aligned} \dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \dots + \dot{x}_n \\ &= \eta - x_1 + \phi_1(x_1) + \dots + \phi_n(x_1) + \theta_1(t)\psi_1(x_1) + \dots + \theta_n(t)\psi_n(x_1), \\ y &= \mu = z_1. \end{aligned}$$

Furthermore

$$\eta\dot{\eta} = \eta(\eta - x_1 + \phi_1(x_1) + \dots + \phi_n(x_1) + \theta_1(t)\psi(x_1) + \dots + \theta_n(t)\psi_n(x_1). \tag{10}$$

Assumption 3.1 $\psi_i(x_1) \leq |x_1|$, $\forall x_1$, $i = 1, 2, \dots, n$.

Case 1 : if $\phi_1(x_1) + \phi_2(x_1) + \dots + \phi_n(x_1) = 0$.
Then

$$\begin{aligned} \eta\dot{\eta} &= \eta^2 - \eta x_1 + \eta\theta_1(t)\psi_1(x_1) + \dots + \eta\theta_n(t)\psi_n(x_1) \\ &\leq \eta^2 - \eta x_1 + |\eta||x_1| \left(|\theta_1(t)| + |\theta_2(t)| + \dots + |\theta_n(t)|\right) \\ &= \eta^2 - \eta\left(\frac{z_1 - \eta}{\alpha - 1}\right) + |\eta|\left|\frac{z_1 - \eta}{\alpha - 1}\right| \left(|\theta_1(t)| + |\theta_2(t)| + \dots + |\theta_n(t)|\right). \end{aligned}$$

Then if $z_1 = 0$ and $0 < \alpha < 1$, we have

$$\eta\dot{\eta} \leq \eta^2 \left(\frac{-\alpha + (|\theta_1(t)| + |\theta_2(t)| + \dots + |\theta_n(t)|)}{|\alpha - 1|}\right). \tag{11}$$

If $|\theta_1(t)| + |\theta_2(t)| + \dots + |\theta_n(t)| < \alpha$, then $\eta\dot{\eta} < 0$. Therefore, the zero dynamics (1) with respect to output μ is asymptotically stable. Thus the system (1) with respect to output μ is minimum phase.

Case 2 : if $\phi_1(x_1) + \phi_2(x_1) + \cdots + \phi_n(x_1) = h(x_1) \neq 0$.
We have

$$\begin{aligned} \eta\dot{\eta} &= \eta(\eta - x_1 + h(x_1) + \theta_1(t)\psi(x_1) + \cdots + \theta_n(t)\psi_n(x_1)) \\ &\leq \eta^2 - \eta x_1 + \eta h(x_1) + |\eta||x_1| \left(|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|\right) \\ &= \eta^2 - \eta\left(\frac{z_1 - \eta}{\alpha - 1}\right) + \eta h\left(\frac{z_1 - \eta}{\alpha - 1}\right) + |\eta|\left|\frac{z_1 - \eta}{\alpha - 1}\right| \left(|\theta_1(t)| + \ldots + |\theta_n(t)|\right). \end{aligned}$$

If $z_1 = 0$, $\forall t$ and $0 < \alpha < 1$, then

$$\eta\dot{\eta} \leq \eta^2 \left(\frac{-\alpha + (|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)|)}{|\alpha - 1|}\right) + \eta h\left(\frac{-\eta}{\alpha - 1}\right). \tag{12}$$

Assumption 3.2 We consider system (1). Choose $\phi_1(x_1)$, $\phi_2(x_1)$, ..., $\phi_n(x_1)$ so that

$$\eta h\left(\frac{-\eta}{\alpha - 1}\right) < 0.$$

If $|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)| \leq \alpha$ and by Assumption 3.2, we have $\eta\dot{\eta} < 0$. Therefore the system (1) with respect to output μ is minimum phase.

Lemma 3.1 *Consider system (1). Then there exists a linear combination of the state variables* $\mu = \alpha x_1 + x_2 + x_3 + \ldots + x_n$ *such that the relative degree of the system (1) with respect to output* μ *is* $n - 1$. *Furthermore due to Assumption 3.1 we obtain*
(i) If $\phi(x_1) + \cdots + \phi_n(x_1) = 0$, *the system (1) with respect to output* μ *is minimum phase, with* $|\theta_1(t)| + |\theta_2(t)| + \ldots + |\theta_n(t)| < \alpha$, $0 < \alpha < 1$.
(ii) If $\phi(x_1) + \cdots + \phi_n(x_1) \neq 0$ *and by Assumption 3.2, the system (1) with respect to output* μ *is minimum phase, with* $0 < \alpha < 1$ *and* $|\theta_1(t)| + \ldots + |\theta_n(t)| \leq \alpha$.

Let μ_d be the desired output of the new output.

Assumption 3.3 Let $x_i = x_{id}$, $i = 1, 2, \ldots, n-2$.

Based on Assumption 3.3, we have x_{2d}, x_{3d}, ..., $x_{(n-1)d}$, respectively. Then $\dot{x}_n = f(x_1, x_{n-1}, x_n)$ can be solved by substituting $x_{n-1} = x_{(n-1)d}$. Thus $x_n = x_{nd}$. Furthermore the definition error $e = \mu - \mu_d$, with $\mu_d = \alpha x_{1d} + x_{2d} + \cdots + x_{nd}$.

We design a control law u in terms of the properties of the solution of higher order ordinary differential equation. Consider a differential equation

$$a_r e^{(r)}(t) + a_{r-1} e^{(r-1)}(t) + \ldots + a_1 \dot{e}(t) + a_0 e(t) = 0, \tag{13}$$

where r is the relative degree of the system. If a polynomial

$$p(s) = a_r s^r + a_{r-1} s^{r-1} + \ldots + a_1 s + a_0 \tag{14}$$

is Hurwitz, then the solution of differential equation (13) tends to zero if $t \to \infty$. In this case, for the purpose of designing the control law, an explicit relationship between input and output is required. To this end, we define a descent function as follows :

$$\begin{aligned} F(\mu, \mu_d, \dot{\mu}, \dot{\mu}_d, \ldots, \mu^{(n-1)}(t), \mu_d^{(n-1)}(t)) &= \left(\sum_{j=0}^{n-1} a_j (\mu - \mu_d)^{(j)}\right)^2 \\ &= \left(\sum_{j=0}^{n-1} a_j (e)^{(j)}\right)^2. \end{aligned} \tag{15}$$