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Output Tracking of Some Class Non-MinimumPhase Nonlinear Systems via linearization Input-Output

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Abstract. We present an output tracking problem for a non-minimum phase nonlinear system. In this paper, the input control design to solve the output tracking problem is to use the input output linearization method. The use of the input output linearization method cannot be initiated from output causing the system to be non-minimum phase. Therefore the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

Keywords: Relative degree of the system, input-output linearization, minimum phase, non-minimum phase

1. Introduction

In the analysis for nonlinear control systems, there is no general method which can be appliedto any nonlinear control system in designing the control input for solving the output tracking problems. Therefore in general, the researchers describe some particular nonlinear classes only. The input-output linearization method is one method that can be used to solve the output tracking problem, but this method is only applicable to minimum phase nonlinear systems, where the relative degree of the system is well-defined [1]. Most of researcher restrict their research to some special nonlinear classes only. In [2], D. Chen and B. Paden have presented a method of stable inversion. The stable inversion method is an iterative tracking for output tracking problems, where the system has an unstable dynamic zero. This method requires that the relative degrees of the system are well-defined and dynamically zero hyperbolic. In [3], Koji Kinosita, et al have discussed iterative learning control using an adjoint system. With iterative learning control, the system tracks the desired output at certain time intervals. Later in [4] also discussed the problem of tracking output for a low-triangular nonlinear system. The control design is through dynamic gain scaling method. In [5], has proposed a control design procedure for tracking output in two steps. the first step is to use input output linearization. The second step is to group some states into internal dynamics as one of the nonlinear subsystems, while the other states become linear subsystems. A nonlinear subsystem is linearized at its equilibrium point. In [6], gradient descent control is used to solve the output tracking problem for a nonlinear system where the unforced system is stable. In [7], S. Baev, et al have discussed the problem of tracking system output for a class of non-minimum phase nonlinear systems using Higher Order Sliding Mode (HOSM). In [8], the output tracking problem is solved by finding the internal dynamic solution of the system. In [9], the issue of tracking output has been discussed at regular time intervals. In [10], J. Naiborhu et.al have discussed the output tracking problem for a non-minimum phase nonlinear class. Input control design begins with redefining the output of the system so that the relative degree of the system is equal to the dimensions of the system. Furthermore, one way to solve the system output tracking problem for a non-minimum phase nonlinear system is to redefine the system output such that the system becomes minimum phase with respect to the new output. research concerning this has been investigated by in [11], [12], [13], [14] In this paper, we will investigate output tracking of some class non-minimum phase nonlinear systems, with relative degree of the system is well defined. For the design of input controls, the system will be transformed through input output linerization. The first step in control design is to redefine the output of the system so that the system is in minimum phase with respect to the new output.

2. Problem formulation

Consider the affine nonlinear control system

$$\dot{x}(t) = f\big(x(t)\big) + g\big(x(t)\big)u \tag{1}$$

$$y(t) = h\big(x(t)\big) \tag{2}$$

where $x(t) \in \mathcal{R}^n, u(t) \in \mathcal{R}. f : D \to \mathcal{R}^n, f(\vec{0}) = \vec{0}$ and $g : D \to \mathcal{R}^n$ are sufficiently smooth in a domain $D \subset \mathcal{R}^n$.

Let a state $y(t) = h(x(t)), h : D \to \mathcal{R}$ is a sufficiently smooth in a domain $D \subset \mathcal{R}^n, h(\vec{0}) = 0$. Let the relative degree of the system (1) with respect to state y is $r, r \le n$. If, the relative degree of the system (1)-(2) is n, the system (1) with respect to state y can be transformed to

$$\dot{z}_k = z_{k+1},\ k = 1, 2,\ \cdots, n-1 \tag{3}$$

$$\dot{z}_k = f(z) + g(z)u \tag{4}$$

$$y = z_1$$

Let the relative degree of the system (1)-(2) is $r, r < n$, the system (1) with respect to state y can be transformed to

$$\dot{z}_k = z_{k+1},\ k = 1, 2,\ \cdots, r-1 \tag{5}$$

$$\dot{z}_k = f(z,\eta) + g(z,\eta)u \tag{6}$$

$$\dot{\eta} = q(z,\eta) \tag{7}$$

$$y = z_1$$

with the internal dynamic

$$\dot{\eta} = q(z,\eta), \tag{8}$$

where $(z,\eta) = (z_1, z_2,\ \cdots, z_r, \eta_1, \eta_2,\ \cdots, \eta_{n-r})$.

If $z_1 = 0$ for all t, the system (8) is said to be zero dynamic with respect to state $y = z_1$. If the zero dynamic with respect to state $y = z_1$ is asymptotically stable, than the system (1)-(2) is minimum phase. [15]

Let $e_i = z_i - y_d^{(i-1)}(t), i = 1, 2, \ldots, \rho$, with y_d is the desired output Then, we have

$$\dot{e}_k = e_{k+1},\ k = 1, 2,\ \cdots, \rho - 1 \tag{9}$$

$$\dot{e}_\rho = a(z,\eta) + b(z,\eta)u - y_d^{(\rho)} \tag{10}$$

$$\dot{\eta} = q(z,\eta) \tag{11}$$

the tracking output problem can be solved by input output linearization technique. The input control which is obtained can be written as a static control

$$u = \frac{1}{b(z,\eta)}\left(-a(z,\eta) + y_d^{(\rho)} - \textstyle\sum_{i=1}^{\rho} c_{i-1} e_i\right) \tag{12}$$

The input control (12), which has variable as solution of internal dynamic system (11). So, The input control (12) can only be used if the system (1)-(2) is minimum phase.

Our objective is to make the output system for a class non-minimum phase tracks the desired output.. Therefore, the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

3. Main results

We will investigate the asymptotic stability for a affine nonlinear control sytem in the following form

$$\dot{x} = Mx + \tau u + \theta(x_1), x(t) \in \mathcal{R}^n, u(t) \in \mathcal{R} \tag{13}$$
$$y = x_1 \tag{14}$$

with $\theta(x_1) \in C^{\infty}(\mathcal{R}^n), \theta(0) = 0, \tau = (0,0,\ldots,0,\tau_{n-1},\tau_n)^T, \tau_{n-1} \neq 0,$

$$\tau_{n-1} = -\tau_n, M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \theta_1(x_1) + \theta_2(x_1) + \cdots + \theta_n(x_1) = 0.$$

The relative of the system (13)-(14) is $n-1$. The system (13)-(14) can be transformed to

$$\dot{\chi}_k = \chi_{k+1},\ k = 1,2,\ \cdots, \tag{15}$$
$$\dot{\chi}_{k+1} = a(\chi,\eta) + b(\chi,\eta)u \tag{16}$$
$$\dot{\eta} = \eta \tag{17}$$
$$y = \chi_1 \tag{18}$$

then the zero dynamic of the system (13)-(14) is

$$\dot{\eta} = \eta$$

Thus the system (13)-(14) is non-minimum phase.
Next, the output of the system (13) will be redefined such that the system (13) will become minimum phase with respect to the new output. We consider system (13). Choose the new output $\beta = \alpha x_1 + x_2 + \cdots + x_n, \alpha \neq 1$.

Let $\beta = \ ax, a = (\alpha,1,\ldots,1), x = (x_1,x_2,\ldots,x_n)^T, \alpha \neq 1$. We have

$$\dot{\beta} = a\dot{x} = aMx + a\theta(x_1) \tag{19}$$
$$\ddot{\beta} = aM^2x + aM\theta(x_1) + a\frac{d\theta}{dt} \tag{20}$$
$$\vdots \tag{21}$$
$$\beta^{(n-2)} = aM^{(n-2)}x + aM^{(n-3)}\theta(x_1) + \cdots + aM\big(\theta(x_1)\big)^{(n-4)} + a\big(\theta(x_1)\big)^{(n-3)} \tag{22}$$
$$\beta^{(n-1)} = aM^{(n-1)}x + aM^{(n-2)}\theta(x_1) + \cdots + aM\big(\theta(x_1)\big)^{(n-3)} + a\big(\theta(x_1)\big)^{(n-2)} + b_n(1-\alpha)u \tag{23}$$

Thus, the relative degree of the system (13) with respectto the state y is $n-1$. Furthermore, the linearized input-state for system (13) with respect to the state $\lambda = z_1$ is

$$\dot{z}_k = z_{k+1},\ k = 1,2,\ \cdots, n-2 \tag{24}$$
$$\dot{z}_{n-1} = f(z,\eta) + g(z,\eta)u \tag{25}$$
$$\begin{aligned}\dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \cdots + \dot{x}_n \\ &= \eta - x_1\end{aligned} \tag{26}$$

with $f(z,\eta) = aM^{(n-1)}x + aM^{(n-2)}\theta(x_1) + \cdots + aM\big(\theta(x_1)\big)^{(n-3)} + a\big(\theta(x_1)\big)^{(n-2)}$,
$g(z,\eta) = b_n(1-\alpha), \alpha \neq 1, z = (z_1,z_2,\ldots,z_{n-1})$.
Futhermore, we will investigate the stabilty of the zero dynamic of the system (13) with respect to the state $v = z_1$. Consider

$$\begin{aligned}\eta\dot{\eta} &= \eta(\eta - x_1) \\ &= \eta^2 - \eta\left(\frac{x_1-\eta}{\alpha-1}\right)\end{aligned} \tag{27}$$

If $z_1 = 0$ and $0 < \alpha < 1$,

$$\eta\dot{\eta} = \frac{\alpha\eta^2}{\alpha-1} < 0 \tag{28}$$

Therefore, the zero dynamic of the system (13) with respect to the state $v = z_1$ is asymptotically stable.

Thus the system (13) with respect to the state $v = z_1$ is minimum phase.
Let v_d is the desired output of the new output.
Assumption : Subtitution $x_i = x_{id},\ i = 1, 2,\ \cdots, n-2$. Based on assumption, we have $x_{2d},\ x_{3d},\ \cdots, x_{(n-1)d}$, respectively. Then $x_{n-1} = f\left(x_1, x_{(n-1)}, x_n\right)$ can be solved by subtituting $x_{(n-1)} = x_{(n-1)d}$
Thus $x_n = x_{nd}$. Furthermore definition error. $e = v - v_d, v_d = \alpha x_{1d} + x_{2d} + \cdots + x_n$
Then, we have

$$\dot{e}_k = e_{k+1},\ k = 1, 2,\ \cdots, n-2 \tag{29}$$

$$\dot{e}_{n-1} = a(z,\eta) + b(z,\eta)u - y_d^{(n-1)} \tag{30}$$

$$\dot{\eta} = q(z,\eta) \tag{31}$$

Based on equation (12) to make the output system (13)-(14) track the desired output, we choose input control

$$u = \frac{1}{b(z,\eta)}\left(-a(z,\eta) + y_d^{(n-1)} - \sum_{i=1}^{n-1} c_{i-1}e_i\right) \tag{32}$$

From equation (32), the system (33)-(34) become

$$\dot{e}_k = e_{k+1},\ k = 1, 2,\ \cdots, n-2 \tag{33}$$

$$\dot{e}_{n-1} = -c_0e_1 - c_0e_1 - \cdots - c_{n-2}e_{n-1} \tag{34}$$

Thus we choose $c_i, i = 0, 1,\ \cdots, n-2$ such that the polynomial

$$p(\lambda) = c_{n-1}\lambda^{n-1} + c_{n-1}\lambda^{n-1} + \ldots + c_1\lambda + c_0 \tag{35}$$

is Hurwitz, then error $e_i(t) \to 0$, if $t \to \infty$. Thus v tend to v_d if time $t \to \infty$. Hence the output of the original system $y = x_1$ track to the desired output $y_d(t)$.

4. Example

Suppose the nonlinear control system is

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2 \\ \dot{x}_2 &= x_3 - u + x_1^2 \\ \dot{x}_1 &= u + 2x_1^2. \\ y &= x_1,\ y_d = sint \end{aligned} \tag{36}$$

The nonlinear system (36)with respect to output y is non-minimum phase. Now, redefinition output $z_1 = v = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$ the relative degree of the system (36) with respect to the state $v = \alpha x_1 + x_2 + x_3$ is 2, $0 < \alpha < 1$. The system (36) with respect to the state v can be transformed to

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= f(z,\eta) + g(z,\eta)u \\ \dot{\eta} &= \eta - \left(\frac{z_1+\eta}{\alpha-1}\right) \end{aligned} \tag{37}$$

with $y = z_1, f(z,\eta) = \alpha x_3 + (\alpha-2)x_1^2 + 2(\alpha-1)x_1x_2 + 2(\alpha-1)x_1^3$,
$g(z,\eta) = 1 - \alpha$.
If $z_1 = 0$, we have

$$\eta\dot{\eta} = \eta\left(\eta\left(\frac{-\eta}{\alpha-1}\right)\right) = \frac{\eta^2\alpha}{\alpha-1} \tag{38}$$

Furthermore if $0 < \alpha < 1$, then $\eta\dot{\eta} < 0$. Thus the system (36) with respect to the state v is minimum phase.
Let $y_d = \sin t$. Next, we choose z_{id} such that if, z_1 track z_{id}, then $y(t)$ track the desired output $y_d(t)$. By replacing x_1 with $x_{1d} = y_d = \sin t$, then $x_{2d} = \cos t - \sin^2(t)$. By replacing x_2 with x_{2d}, we have a differential equation $\dot{x}_3 - x_3 = \sin(t) + \sin(2t) - \sin^2(t)$. Thus $x_{3d} = -0.5\cos(t) - 0.5\sin(t) - cos^2(t) + 1$. Now, $z_{1d} = \alpha x_{1d} + x_{2d} + x_{3d} = (\alpha - 0.5)\sin(t) + 0.5\cos(t)$. According to (32), the input control is

$$u = \frac{1}{1-\alpha}\left(-f(z,\eta) + z_{1d}^{(2)} - c_0e_1 - c_1e_2\right), \tag{39}$$

with $z_{1d}^{(2)} = (0.5 - \alpha)\sin(t) - 0.5\cos(t), e_1 = z_1 - z_{1d}, e_2 = e_1$.

The simulation result are shown in figure 1 and figure 2

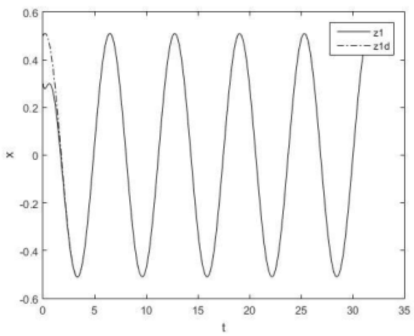

Figure 1. Output tracking z_1 to z_{1d}.

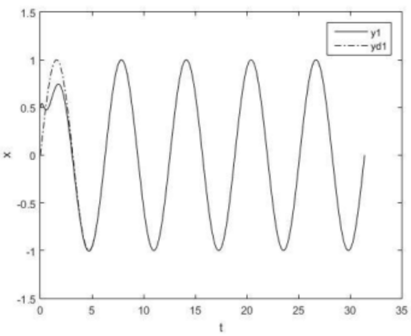

Figure 2. Output tracking y to yd.

5. Conclusions

In this paper, we have investigated output tracking for a non-minimum phase nonlinear system (13)-(14) using input controls. The control design is using the input output linearization method. To apply the input-output linearization method, the system output (13) is redefined so that the system (13) becomes minimum phase with respect to the new output, where the new output is linear combination of the state variables. By setting a certain assumption, the new desired output will be set based on the desired output of the original system.

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