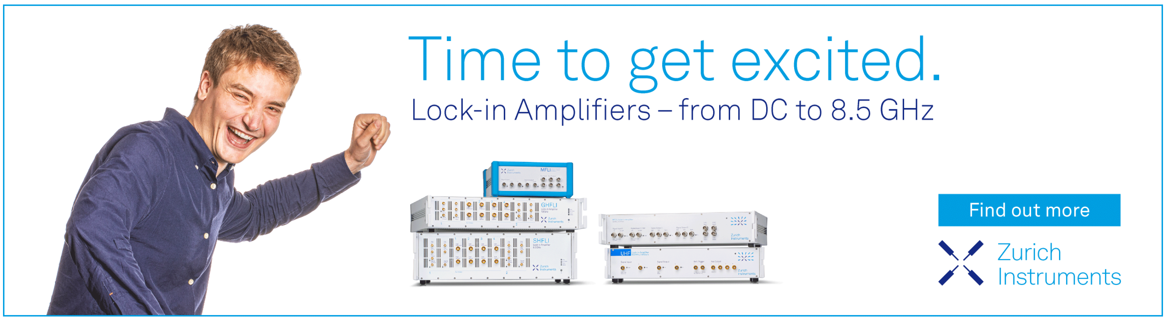
AIP Conference Proceedings

RESEARCH ARTICLE | FEBRUARY 29 2016

Output tracking of some class non-minimum phase nonlinear systems via output redefinition

Firman; Janson Naiborhu

AIP Conference Proceedings 1716, 020003 (2016)
https://doi.org/10.1063/1.4942986



Output Tracking of Some Class Non-Minimum Phase Nonlinear Systems Via Output Redefinition

Firman, Janson Naiborhu

Industrial & Financial Mathematics Group, ITB

Abstract. In this paper, we present the output tracking for a class non-minimum phase nonlinear. To achive the output tracking, we will apply the modified steepest descent control. To apply the modified steepest descent control, the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

INTRODUCTION

In the output tracking theory, the input output linearization is one of the most available methods [1]. Output tracking problems for nonlinear non-minimum phase systems is a rather difficult issue in control theory. Most of researcher restrict their research to some special nonlinear classes only. The stable inversion proposed in [2], [3] is an iterative solution to the tracking problem with the unstable zero dynamics. This method requires the system to have well defined relative degree and hyperbolic dynamics, i.e. no eigenvalues on the imaginary axis. In [4], control design procedure for the output tracking was proposed. The design procedure consists of two steps. In the first step, the standard input output linearization is applied. In the second step, we group an output with the internal dynamics as one subsystems, which is usually nonlinear, and the rest of the output as the other subsystem that is linear, the nonlinear subsystems is linearized about its equilibrium. In [5], the asymptotic output tracking which is a class of causal nonminimum phase uncertain nonlinear systems is achieved by using higher order sliding modes (HOSM) without reduction of the input-output dynamics order. In [6], J. Naiborhu *et.al* have developed a method to design the input control to track the output of a non-minimum phase nonlinear systems asymptotically. The design of the input control is based on the exact linearization. To perform an exact linearization, the output should be selected such that its relative degree is equal to the dimension of the system. Results on stabilization of non-minimum phase system in the output feedback form have been presented in In [7], [8], [9]. The main idea in [7], [8], [9] is output reconstruction such that becomes minimum phase with respect to a new output.

In this paper, we will modify the steepest descent control for output tracking of a class non-minimum phase nonlinear uncertain systems, with relative degree being $n-1$, n is the dimension of the system. The modification is the addition of an input artificial of the steepest descent control. The design of descent control can not be initiated from the output causing the system to be non-minimum phase. In this paper to solve the problem, we transform the system into a normal form which is minimum phase with respect to a virtual output, which is a linear combination of state variables.

Problem Statement

Consider affine nonlinear system

$$\dot{x} = \mathbf{A}x + bu + \phi(y), \; x(t) \in \mathbf{R}^n, \; u(t) \in \mathbf{R} \tag{1}$$

$$y = x_1 \tag{2}$$

Application of Mathematics in Industry and Life
AIP Conf. Proc. 1716, 020003-1–020003-7; doi: 10.1063/1.4942986

in which $\phi(y)$ is smooth vector field in $\mathbf{R}^n$, with $\phi(0) = 0$, $b = [0, \dots, 0, b_{n-1}, b_n]^T$, $b_{n-1} \neq 0$, $b_{n-1} = -b_n$, $\mathbf{A} = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix}$

The relative degree of the system (1)-(2) is $n-1$. The system (1)-(2), can be transformed to

$$\begin{aligned} \dot{\omega}_k &= \omega_{k+1}, \qquad k = 1, \dots, n-2 && (3) \\ \dot{\omega}_{n-1} &= a(\omega\ ,\ \eta)\ +\ b(\omega\ ,\ \eta)u && (4) \\ \dot{\eta} &= \eta - x_1 + \phi_1(x_1) + \cdots + \phi_n(x_1) && (5) \\ y &= \omega_1. \end{aligned}$$

Then the zero dynamic of the system (1)-(2) is

$$\dot{\eta} = \eta.$$

Thus the system (1)-(2) is non-minimum phase.
Our objective is to make the output system (1)-(2) tracks the desired output. To make the system (1)-(2) track the desired output, we will use the dynamic feedback control. The design of the dynamic control is based on the modification of the steepest descent control. By ”Trajectory Following Method [10], the steepest descent control is determined from the differential equation $\dot{u} = -\frac{\partial F}{\partial u}$, where F is a descent function which has a variable as solution of internal dynamics system. So, to modify the steepest descent control can not be initiated from the output causing the system to be non-minimum phase. Therefore, the output of the system will be redefined such that the system will become minimum phase with respect to a new output.

Output Tracking

We consider system (1). Consider now a new output $\mu = \alpha x_1 + x_2 + \cdots + x_n$. The relative degree of system (1) with respect to μ is $n-1$. The system (1) with respect to μ, can be transformed to

$$\begin{aligned} \dot{z}_k &= z_{k+1}, \qquad k = 1, \dots, n-2 && (6) \\ \dot{z}_{n-1} &= a(z\ ,\ \eta)\ +\ b(z\ ,\ \eta)u && (7) \\ \dot{\eta} &= \dot{x}_1 + \dot{x}_2 + \dots + \dot{x}_n \\ &= \eta - x_1 + \phi_1(x_1) + \dots + \phi_n(x_1) \\ \mu &= z_1. \end{aligned}$$

Furthermore

$$\eta\dot{\eta} = \eta(\eta - x_1 + \phi_1(y) + \cdots + \phi_n(y)) \tag{8}$$

Case 1 : if $\phi_1(y) + \cdots + \phi_n(y) = 0$.
Then

$$\begin{aligned} \eta\dot{\eta} &= \eta(\eta - x_1) = \eta^2 - \eta x_1 \\ &= \eta^2 - \eta(\frac{z_1 - \eta}{\alpha - 1}) \end{aligned} \tag{9}$$

Then if $z_1 = 0$ and $0 < \alpha < 1$, then

$$\eta\dot{\eta} = \frac{\alpha\eta^2}{\alpha - 1} < 0. \tag{10}$$

Therefore, the zero dynamic (1) with respect to output μ is asymptotic stable. Thus the system (1) with respect to output μ is minimum phase.
Case 2 : if $\phi_1(y) + \cdots + \phi_n(y) = h(y) \neq 0$.
Then

$$\eta\dot{\eta} = \eta(\eta - x_1 + h(y)) = \eta(\eta - \left(\frac{z_1 - \eta}{\alpha - 1}\right) + h\left(\frac{z_1 - \eta}{\alpha - 1}\right)$$

if $z_1 = 0$ and $0 < \alpha < 1$, then

$$\eta\dot{\eta} = \frac{\alpha\eta^2}{\alpha - 1} + \eta h\left(\frac{-\eta}{\alpha - 1}\right) \tag{11}$$

If we choose $h(y)$ such that $\eta h\left(\frac{-\eta}{\alpha-1}\right) < 0$. then the zero dynamic (1) with respect to output μ is asymptotic stable

Let μ_d is the desired output of the new output.
Assumption : Substituion $x_i = x_{id}$, $i = 1, 2, \ldots, n-2$.
Based on assumption, we have $x_{2d},\ x_{3d},\ \ldots,\ x_{(n-1)d}$, respectively.
Then $\dot{x}_n \ = \ f(x_1, x_{n-1}, x_n)$ can be solved by substituting $x_{n-1} = x_{(n-1)d}$.
Thus $x_n = x_{nd}$. Furthermore definition error

$$e = \mu - \mu_d,\ \ \mu_d = \alpha x_{1d} + x_{2d} + \cdots + x_{nd}$$

. We design a control law u through properties of the solution of higher order ordinary differential equation. Consider a differential equation

$$a_r e^{(r)}(t) + a_{r-1}e^{(r-1)}(t) + \ldots + a_1\dot{e}(t) + a_0 e(t) = 0, \tag{12}$$

where r is the relative degree of the system. If a polynomial

$$p(s) = a_r s^r + a_{r-1}s^{r-1} + \ldots + a_1 s + a_0 \tag{13}$$

is Hurwitz, then solution of differential equation (12) tends to zero if $t \to \infty$.
In this case for the purpose of designing the control law required an explicit relationship between input and output. For that, we define a descent function as follow :

$$\begin{aligned} F(\mu, \mu_d, \dot{\mu}, \dot{\mu}_d, \ldots, \mu^{(n-1)}(t), \mu_d^{(n-1)}(t)) &= \left(\sum_{j=0}^{n-1} a_j(\mu - \mu_d)^{(j)}\right)^2 \\ &= \left(\sum_{j=0}^{n-1} a_j(e)^{(j)}\right)^2. \end{aligned} \tag{14}$$

By"Trajectory Following Method" [10], the control u is determined from the differential equation

$$\dot{u} = -\frac{\partial F}{\partial u} = -2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e_1)^{(j)}\right)\frac{\partial(e_1)^{(n-1)}}{\partial u}, \tag{15}$$

The control law in equation (15) is called the steepest descent control.
Calculate the time derivative of the descent function (14) along the trajectory of the extended system

$$\dot{x} \ = \ Ax + bu + \phi(y), \tag{16}$$

$$\dot{u} \ = \ -2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e_1)^{(j)}\right)\frac{\partial(e_1)^{(n-1)}}{\partial u} \tag{17}$$

Then we have

$$\begin{aligned} \dot{F}(e_1, \dot{e_1}, \ldots, e_1^{(r)}) &= 2\left(\sum_{j=0}^{n-1} a_j(e_1)^{(j)}\right)\left(\sum_{j=0}^{n-2} a_j(e_1)^{(j+1)}\right) \\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e_1)^{(j)}\right)\left(\frac{\partial a(e+\beta_d, \eta)}{\partial t} + \frac{\partial b(e+\beta_d, \eta)}{\partial t}u\right) \\ &- 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j(e_1)^{(j)}\right)y_d^{(n)} - \left(\frac{\partial F}{\partial u}\right)^2. \end{aligned} \tag{18}$$

From equation (18), the value of the time derivative of the descent function (14) along the trajectory of (16)- (17) can not be guaranteed to be less than zero $t \geq 0$.
Now we modify the steepest descent control (15) by adding an artificial input v. Thus $\dot{u} = -\frac{\partial F}{\partial u} + v$
By the same way, let us calculate the time derivative of the descent function (14) along the trajectory of the extended system yielding

$$\begin{aligned} \dot{F}(e_1, \dot{e}_1, \dots, e_1^{(r)}) &= 2\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)\left(\sum_{j=0}^{n-2} a_j (e_1)^{(j+1)}\right) \\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)\left(\frac{\partial a(e+\beta_d,\eta)}{\partial t} + \frac{\partial b(e+\beta_d,\eta)}{\partial t}u\right) \\ &- 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)y_d^{(n)} - \left(\frac{\partial F}{\partial u}\right)^2 + \frac{\partial F}{\partial u}v \end{aligned} \tag{19}$$

We will choose the artificial input v such that $\dot{F}(e_1, \dot{e}_1, \dots, e_1^{(r)})$ be less then zero. If we take

$$v = \frac{1}{\frac{\partial F}{\partial u}}\left(-k(e_1, \dot{e}_1, \dots, e_1^{(n-1)}) - \sqrt{\left(k\left(e_1, \dot{e}_1, \dots, e_1^{(n-1)}\right)\right)^2 + \left(\frac{\partial F}{\partial u}\right)^2}\right), \tag{20}$$

where

$$\begin{aligned} k(e_1, \dot{e}_1, \dots, e_1^{(n-1)}) &= 2\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)\left(\sum_{j=0}^{n-2} a_j (e_1)^{(j+1)}\right) \\ &+ 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)\left(\frac{\partial a(e+\beta_d,\eta)}{\partial t} + \frac{\partial b(e+\beta_d,\eta)}{\partial t}u\right) \\ &- 2a_{n-1}\left(\sum_{j=0}^{n-1} a_j (e_1)^{(j)}\right)y_d^{(n)}. \end{aligned} \tag{21}$$

Then

$$\dot{F}(e_1, \dot{e}_1, \dots, e_1^{(n-1)}) = -\left(\frac{\partial F}{\partial u}\right)^2 - \sqrt{\left(k\left(e_1, \dot{e}_1, \dots, e_1^{(n-1)}\right)\right)^2 + \left(\frac{\partial F}{\partial u}\right)^2} \tag{22}$$

We have $\dot{F}(e_1, \dot{e}_1, \dots, e_1^{(n-1)}) < 0$, if $\sum_{j=0}^{n-1} a_j (e_1)^{(j)} \neq 0$. Thus, the descent function (14) becomes minimum. The minimum value of descent function (14) is zero. Therefore $F(e_1, \dot{e}_1, \dots, e_1^{(n-1)}) = 0$, then $\sum_{j=0}^{n-1} a_j (e_1)^{(j)} = 0$. Thus, we choose a_j, $j = 0, \dots, n-1$ such that the polynomial $p(s) = a_0 + a_1 s + \cdots + a_{r-1}s^{n-2} + s^{n-1}$ is Hurwitz, then error $e_1(t) \to 0$, if $t \to \infty$. Thus μ tend to μ_d if time $t \to \infty$. Hence the output of the original system $y = x_1$ tracks to the desired output $y_d(t)$.

Example

Example 1. Consider nonlinear system (SISO)

$$\begin{aligned} \dot{x_1} &= x_2 + x_1^2 \\ \dot{x_2} &= x_3 - u + x_1^2 \\ \dot{x_3} &= u - 2x_1^2 \\ y &= x_1, \quad y_d = sin(t) \end{aligned} \tag{23}$$

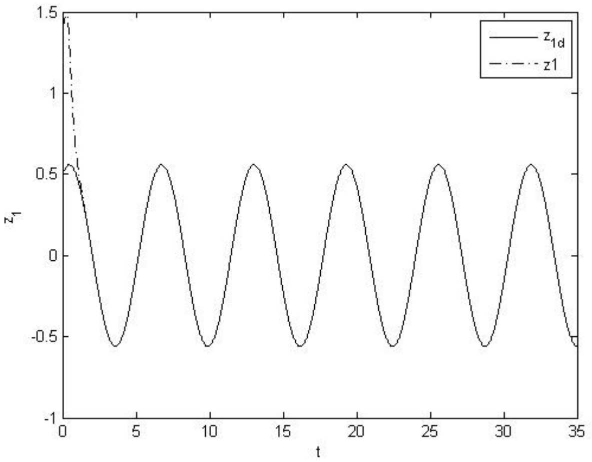


FIGURE 1. output tracking z_1 *to* z_{1d}

The nonlinear system (23) with respect to output y is non-minimum phase. Now, redefinition output $z_1 = \mu = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$.

By considering the new output, the relative of the system (23) is 2 and normal form

$$\begin{aligned} \dot{z_1} &= z_2 \\ \dot{z_2} &= b(z) + a(z)u \\ \dot{\eta} &= \eta - \left(\frac{z_1 - \eta}{\alpha - 1}\right), \end{aligned} \tag{24}$$

with $a(z) = \alpha x_3 + (\alpha - 2)x_1^2 + 2(\alpha - 1)x_1 x_2 + 2(\alpha - 1)x_1^3$, $b(z) = 1 - \alpha$.

If $z_1 = 0$, we have

$$\eta\dot{\eta} = \eta\left(\eta - \left(\frac{-\eta}{\alpha - 1}\right)\right) = \frac{\eta^2\alpha}{\alpha - 1},$$

Furthermore if $0 < \alpha < 1$, then $\eta\dot{\eta} < 0$. Thus system (23) in respect to output z_1 is minimum phase.

Let $y_d(t) = sin(t)$. Next, we choose z_{1d} such that if z_1 track $z_{1d}(t)$, then $y(t)$ tracks the desired output $y_d(t)$. By replacing x_1 with $x_{1d} = y_d = sin(t)$, then $x_{2d} = cos(t) - sin^2(t)$. By replacing x_2 with x_{2d}, we have a differential equation $\dot{x_3} - x_3 = sin(t) + sin(2t) - sin^2(t)$.

Thus $x_{3d} = -0.5cos(t) - 0.5sin(t) - cos^2(t) + 1$. Now, $z_{1d} = \alpha x_{1d} + x_{2d} + x_{3d} = (\alpha - 0.5)sin(t) + 0.5cos(t)$. According to (15), the modified steepest descent control is

$$\dot{u} = -2a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z_1} - \dot{z_{1d}}) + a_2(\ddot{z_1} - \ddot{z_{1d}}))(1 - \alpha) + v, \tag{25}$$

with v as in (20).

The simulation results are shown in figure 1 and figure 2.

Example 2.

$$\begin{aligned} \dot{x_1} &= x_2 - x_1^3 \\ \dot{x_2} &= x_3 - u + 2x_1^3 \\ \dot{x_3} &= u - x_1^3 \\ y &= x_1, \quad y_d = sin(t) \end{aligned} \tag{26}$$

The nonlinear system (26) has relative degree 2. The system (26) is non-minimum phase. Now redefinition output : $z_1 = \mu = \alpha x_1 + x_2 + x_3$, with $0 < \alpha < 1$. Furthermore

$$\dot{z_1} = \alpha x_2 + x_3 - \alpha x_1^3$$

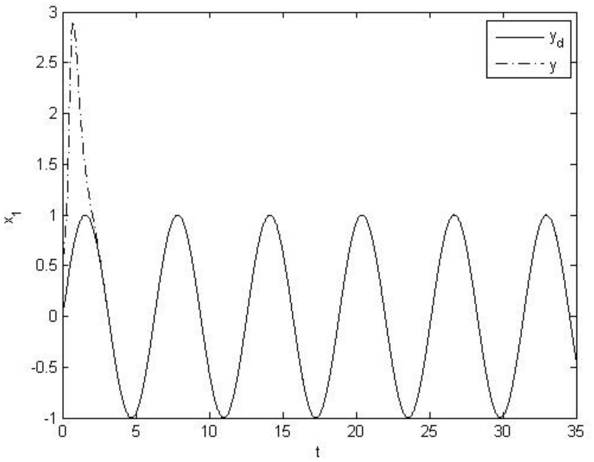


FIGURE 2. output tracking $y \ \ to \ \ y_d$

$$\ddot{z}_1 = \alpha\dot{x}_2 + \dot{x}_3 - 3\alpha x(1)^2\dot{x}_1$$

Thus the relative of the systems (26) with respect to the output z_1 is 2.
If $z_1 = 0$, we have

$$\eta\dot{\eta} = \frac{\alpha\eta^2}{\alpha - 1} + \frac{\eta^4}{(\alpha - 1)^3} \tag{27}$$

Therefore system (26) is minimum phase with respect to a new output.
Let $y_d(t) = sin(t) = x_{1d}(t)$. Next, we chose $z_{1d}(t)$ such that if $z_1(t)$ tracks $z_{1d}(t)$, then $y(t)$ tracks the desired output $y_d(t)$. By replacing x_1 with $x_{1d}(t) = sin(t)$, then we have $x_{2d} = cos(t) + sin^3 t$. By replacing x_2 with $x_{2d}(t)$, we have a differential equation $\dot{x}_3 - x_3 = sin(t) - 3sin^2 tcost$.
Thus $x_{3d} = 1/10cos(t) - 1/5sin(t) - 3/10cos^3 t - 9/10sin^3 t$. Next, $z_{1d} = x_{1d} + 2x_{2d} + 2x_{3d}$. According to (15), the modified steepest descent control is

$$\dot{u} = -2x_2a_2(a_0(z_1 - z_{1d}) + a_1(\dot{z}_1 - \dot{z}_{1d}) + a_2(\ddot{z}_1 - \ddot{z}_{1d})) + v, \tag{28}$$

with v as in (20). The simulation results are shown in figure 3 and figure 4

Conclusions

In this paper, we have investigated the output tracking for a class non-minimum phase nonlinear systems (1)-(2). The modified steepest descent control has been applied for the output tracking. To apply the modified steepest decent control the system (1) are required to be minimum phase with respect to a new output, where the new output is linear combination of the state variables. Furthermore, the new desired output will be set based on the desired output of the original system.

REFERENCES

[1] A. Isidori. *Nonlinear Control Systems: An Introduction*. Secon Edition Springer-Verlag Berlin, Heidelberg 1989.

[2] D.Chen and B.Paden (1996). Stable inversion of nonlinear non-minimum Phase systems, *Int.J.Control* , Vol.64,No.1, 45-54.

[3] D.Chen (1993). Iterative solution to stable inversion of nonlinear non-minimum Phase systems, *Proc. American Control Conference*, June, 2960-2964.

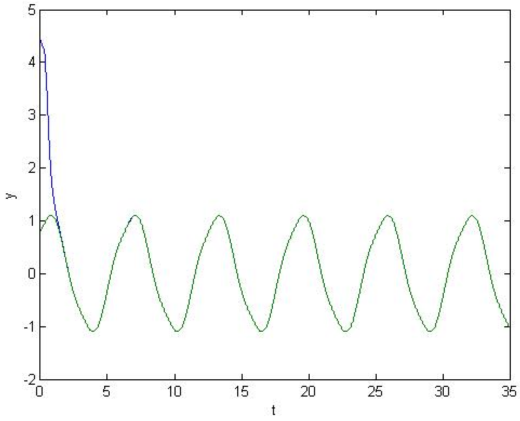


FIGURE 3. output tracking z_1 *to* z_{1d}

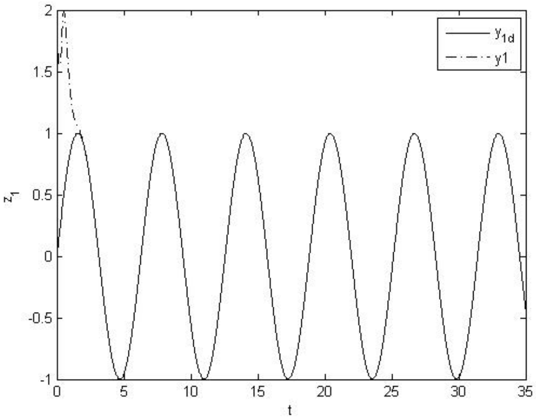


FIGURE 4. output tracking y *to* y_d

[4] Dong Li. (2005). Output Tracking of Nonlinear Nonminimum Phase Systems : an Enginering Solution, *Proceedings of the 44th IEEE Conference on Decision and control, and the European Control Conference*, Sevilla,Spain,Dec 12-15, 3462-3467.

[5] S. Baev, Y.Shtessel, I. Shkolnikov. (2007). HOSM driven outputvtracking in the nonminimum-phase causal nonlinear Systems, *Proceeding of the 46th IEEE Conference on Decision and Control*, New Orleands,LA,USA, Des 12-14, 3715-3720.

[6] J. Naiborhu, Firman, K, Mu'tamar. *Particle swarm optimization in the exact linearization technic for output tracking of non-minimum phase nonlinear systems* Appl. Math Sci 7 (109), 2013, 5427-5442.

[7] Riccardo Marino, Patrizio Tomei. (2004). A class of Globally Output Feedback Stabilizable Nonlinear Nonminimum Phase Systems, *43rd IEEE Conference on Decision and Control*, Atlantis, Paradise Island,Bahamas Des 14-17, 4909-4914.

[8] Z.Y.Zili Li Zengqiang Chen. *The stability analysis and control of nonminimum phase systems*. Int. J. Nonlinear Sci. 3(2), 2007, 103-110.

[9] Firman, Janson Naiborhu, Roberd Saragih. *Modification of a steepest descent control for outout tracking of some class non-minimum phase nonlinear systems*. Applied Mathematics and Computation 269 (2015) 497-506.

[10] T.L. Vincent, W.J. Grantham. *Nonlinear and Optimal Control Systems*. John Wiley and Sons, Inc, Newyork 1997.