

Likelihood Construction of Hawkes Process in Insurance Claim Settlement Process

by

FILE	ILYAS_2019_J._PHYS._CONF._SER._1341_092016.PDF (599.51K)		
TIME SUBMITTED	23-NOV-2019 07:35PM (UTC+0700)	WORD COUNT	1583
SUBMISSION ID	1220120482	CHARACTER COUNT	8412

PAPER • OPEN ACCESS

Likelihood Construction of Hawkes Process in Insurance Claim Settlement Process

To cite this article: N Ilyas *et al* 2019 *J. Phys.: Conf. Ser.* **1341** 092016

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the collection - download the first chapter of every title for free.

Likelihood Construction of Hawkes Process in Insurance Claim Settlement Process

N Ilyas, N Sunusi, Anisa and St Sahrman

Department of Statistics, Faculty of Mathematics and Natural Sciences, Hasanuddin University, Jl. Perintis Kemerdekaan Km.10 Tamalanrea, Makassar-Indonesia

ntitisanusi@gmail.com

Abstract. The construction of the likelihood equation in the point process models plays a very important role, especially to obtain a good estimator of the parameters of a model. This paper aims to construction of the likelihood of Hawkes process in solving the problem of the claim filing process in the nonlife insurance company. In the construction process, the excitation function with exponential decay is used. The result shows that the total likelihood function of the process depend on probability no claim at time t , the number of policyholder at time t , and the excitation function.

1. Introduction

Hawkes process is one of simple point process that is often used in studies of seismology, Deoxiribo Nucleat Acid modeling, finance [1], insurance problem [2], [3]. In 2010, Stabile and Torrisi [2] have examined the possibility of ruin problems using Hawkes processes. In insurance premium matters, another important thing is the determination of premium in general as described by Jang and Dassios & Dassios, Jang and Dassios & Dassios, in 2012 [4]. In 1998, Arjas studied about claim process estimation through regression model Zero-Inflated Poisson [5].

The use of Point process models in the claim submission process assumes claims as points on a certain time interval. The occurrence of claim settlement is explained completely by a stochastic process, where two claims that occur at different times occur in sequence as was done by Nurtiti et al 2016 [3]. This study examines a model point process in discussing to calculate claim reserves. The determination of claim reserves discussed by Nurtiti et al uses the assumption that the time of submission of each claim is a random variable.

Until now, the problem of submitting claims to insurance companies is still being studied because it is not easy to predict. This is because the time for filing claims from each event is random. Besides that, submission of claims in insurance companies, including recurring events, but the frequency is not the same the number of accidents that occur in everyday life. In the study of mathematical modeling, models that can explain events like this are known as point processes. Thus, the process of submitting claims to insurance companies can be described through point process modeling. This study aims to construct the likelihood equation model of claims settlement process for Hawkes process with the excitation function is exponential decay.

2. Hawkes Process

Hawkes process is a process class where the arrival rate of events is depend on past events - that is, an self exciting process and next we will examine in detail the most famous exciting process, that is Hawkes process [6].

Definition 1. Poisson Process

Let (T_1, T_2, \dots) be a sequence of random variables that has the value in $[0, +\infty)$, with $P(0 \leq T_1 \leq \dots) = 1$, and the number of points in a bounded region is almost surely finite, then the sequence of (T_1, T_2, \dots) is called a point process [7].

Definition 2. One Dimensional Hawkes Process

Let $N(t)$ be a point process with characterized by

$$\lambda(t) = \lambda + \int_0^t \mu(t-s) dN(s) \quad (1)$$

where $\lambda(t)$ is intensity, so it is called the one dimensional Hawkes process where $\lambda > 0$, and the response function $\mu(t)$ is a positive function and satisfies $\int_0^{+\infty} \mu(s) ds < 1$.

3. Likelihood Function of Point Process Model

Let $N(t)$ be a point process on $[0, T]$ for $T < \infty$ and let $\{T_1, T_2, \dots, T_n\}$ be a realization of event times of $N(t)$ over the period $[0, T]$. The data likelihood L defined as a parameter function set θ as follows [7]:

$$L(\theta) = \prod_{i=1}^n \lambda(T_i) e^{-\int_0^{T_i} \lambda(t) dt} \quad (2)$$

Based on likelihood formula along the lines, at some time t , the history \mathcal{H}_t is the histories of times events T_1, T_2, \dots, T_n up to but not including time t . We define $f^* := f(t|\mathcal{H}_t)$ as the function of conditional probability density of time t of the next event T_{n+1} given the history of previous event T_1, T_2, \dots, T_n . Define that $P\{T_{n+1} \in (t, t + dt)\} = f_{T_{n+1}}(t) dt$. We have

$$f(T_1, T_2, \dots, T_n) = \prod_{i=1}^n f(T_i | T_1, T_2, \dots, T_{i-1}) \prod_{i=1}^n f^*(T_i) \quad (3)$$

The event intensity $\lambda(t)$ is correspondence to terms of the conditional density f^* and its cumulation distribution function F^* [8].

The Hawkes process has clustering effect, self-exciting property, and long memory. This is reflected in the conditional intensity function:

$$\lambda^*(t) = \lambda + \int_0^t \mu(t-s) dN(s) \quad (4)$$

where $\lambda > 0$ be the background intensity, $\mu > 0$ is the function of excitation describing how much the intensity affected by past jumps.

The maximum likelihood estimation will be very effective and its asymptotic normality, consistency, and efficiency were proved in Ogata (1978) [9].

4. Methodology

To construct of likelihood equation of Hawkes process for claim process, the exponential decay excitation function is used. We choose an excitation function, namely the most commonly used exponential decay:

$$\mu(t) = \alpha e^{-\beta t} \quad (5)$$

Thus, from now on our conditional intensity function will be:

$$\lambda^*(t) = \lambda + \alpha \sum_{\tau_i < t} e^{\beta(t-\tau_i)} \quad (6)$$

Thus, each arrival makes the intensity immediately jump up by α and over time the impact of the arrival decays exponentially at rate β .

5. Result

Likelihood Construction of Hawkes Process

Let θ be the parameters of Hawkes Process. Maximum likelihood estimation of the Hawkes Process can be found by maximizing the function of likelihood in equation (2). The maximum likelihood estimate $\hat{\theta}$ is defined to be $\hat{\theta} = \arg \max(\theta)$.

Let $N(t)$ is a point process on $[0, T]$ for $T < \infty$ and $\{T_1, T_2, \dots, T_n\}$ is the set of event times of $N(t)$ in time interval $[0, T]$. The likelihood L of data as equation (2) denoted by:

$$L(\theta) = \prod_{i=1}^n \lambda(T_i) e^{-\int_0^T \lambda(t) dt} \quad (7)$$

The event intensity $\lambda(t)$ can be denoted in terms of the function of conditional density f^* and as correspondence with cumulative distribution function F^* [8]:

$$\lambda(t) = \frac{f^*(t)}{1 - F^*(t)} \quad (8)$$

In this study a likelihood equation will be constructed that can be used to estimate the parameters of the claim filing model. Suppose that t_i is the time of filing a claim- i , t_{i+1} is time of claim occurrence at the time to $i + 1$, and T is a random variable that denote time of claim occurrence. The likelihood function for the occurrence claim- i is a probability density function for submission of claims at a certain time if it is known that there are no claim until time t , so that for occurrence a claim at time t_i , we obtained

$$L_i = f(t_i | T > t) = \frac{d}{dt_i} F(t_i | T > t) = \frac{f(t_i)}{1 - F(t_i)} = \frac{f(t_i)}{S(t_i)} \quad (9)$$

Equation (*) is contribution of likelihood equation (L) to filing claim- i th, $\lambda(t_i)$ is hazard rate at time t_i . When we assume that $s_i = t_i - t$ is time of claim- i th occurrence in time interval $(t, t+1]$ where $0 < s_i \leq 1$, then

$$L_i = \frac{S(t+s_i)\lambda(t+s_i)}{S(t)} \quad (10)$$

We know that $\frac{S(t+s_i)}{S(t)} = \frac{\exp(-\int_0^{t+s_i} \mu(t) dt)}{\exp(-\int_0^t \mu(t) dt)}$ denote probability that no claim occurs in interval $(t, t + s_i]$, and $\lambda(t + s_i)$ is hazard rate of filing claim at time $+s_i$. Thus, likelihood function for all time of claim filing is $\prod_{i=1}^n \frac{\exp(-\int_0^{t+s_i} \mu(t) dt)}{\exp(-\int_0^t \mu(t) dt)} \cdot \lambda(t + s_i)$.

In claim settlement process, total likelihood function is number of policyholder that no claim filing process at time t denoted by $(n - d)$ in probability no claim process $p_t^{n-d} = (1 - q_t)^{n-d}$, where $p_t + q_t = 1$ and d is the number of claim process at time t , n_t is the number of policyholder at time t , and p_t is probability no claim at time t denoted by $1 - q_t$, so that total likelihood function is:

$$L(\theta) = (1 - q_t)^{n-d} \prod_{i=1}^n \frac{\exp(-\int_0^{t+s_i} \mu(t) dt)}{\exp(-\int_0^t \mu(t) dt)} \cdot \lambda(t + s_i) \quad (11)$$

So the total likelihood function of Hawkes model for claim occurrence is:

$$L(\theta) = (1 - q_t)^{n-d} \prod_{i=1}^n \frac{\exp(-\int_0^{t+s_i} \mu(t) dt)}{\exp(-\int_0^t \mu(t) dt)} [\lambda + \alpha \sum_{i=1}^n e^{\beta(t-t_i)}] \quad (12)$$

6. Conclusion

The total likelihood equation in the claim settlement process in an insurance company plays a very important role. Therefore, accurate likelihood construction is needed. Likelihood function of Hawkes model for claim occurrence with exponential decay excitation function shows that the total likelihood

function of the process depend on probability no claim at time t , the number of policyholder at time t , and the excitation function.

7. Acknowledgement

The authors would like to acknowledge to research grant of Hasanuddin University who has support the implementation of this research.

References

- [1] Embrechts, P., Liniger, T. and Lin, L. (2011): Multivariate Hawkes Processes: An Application to Financial Data. *Journal of Applied Probability*, 48A, 367-378.
- [2] Stabile, G. and G. L. Torrisi. (2010): Risk processes with nonstationary Hawkes arrivals. *Methodol. Comput. Appl. Prob.*, 12, 415-429.
- [3] Nurtiti Sunusi , Aidawayati R, Irmayani, 2016. Study of Insurance Claim using Point Process Models, *Indian Journal of Science and Technology*, 29(8).
- [4] Dassios, A. and Jang, J. (2012): A Double Shot Noise Process and Its Application in Insurance. *J. Math. System Sci.*, 2, 82-93.
- [5] Arjas E. The claims reserving problem. In *non-life insurance: Some structural ideas*. *Astin Bulletin*. 1998; 19(2):139–52.
- [6] Hawkes, A.G. (1971). Point spectra of some mutually exciting point processes. *J. Roy. Statist. Soc. Ser. B Stat. Methodol.*, pp.438-443.
- [7] Daley DJ, Vere-Jones D. An introduction to the theory of point processes. *Elementary Theory and Methods*. 2nd ed. New York: Springer; 2003.
- [8] Rasmussen, J, 2011. Temporal Point Process: the conditional Intensity Function.
- [9] Rasmussen. J.G. 2013. Bayesian inference for Hawkes processes. *Methodology and Computing in Applied Probability*, 15(3): 623–642. ISSN 1387-5841.
- [10] Ogata, Y. (1978). Statistical models for earthquake occurrences and residual analysis for point processes. *J. Amer. Stat. Assoc.*, 83(401), 9-27 .

Likelihood Construction of Hawkes Process in Insurance Claim Settlement Process

ORIGINALITY REPORT

% **17**
SIMILARITY INDEX

% **12**
INTERNET SOURCES

% **9**
PUBLICATIONS

% **5**
STUDENT PAPERS

PRIMARY SOURCES

1 www.i-scholar.in Internet Source % **6**

2 arxiv.org Internet Source % **5**

3 Marian-Andrei RizoIU, Young Lee, Swapnil Mishra. "Hawkes processes for events in social media", Association for Computing Machinery (ACM), 2017 Publication % **2**

4 Anatoliy Swishchuk. "Risk Model Based on Compound Hawkes Process", Wilmott, 2018 Publication % **1**

5 Submitted to University of Limerick Student Paper % **1**

6 link.springer.com Internet Source % **1**

7 Submitted to Cardiff University Student Paper % **1**

8

Submitted to University of Warwick

Student Paper

% 1

9

www.ets.org

Internet Source

% 1

10

Submitted to Chiang Mai University

Student Paper

% 1

EXCLUDE QUOTES ON

EXCLUDE ON

BIBLIOGRAPHY

EXCLUDE MATCHES

< 5

WORDS