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The Binomial Model for N-Period European Call Option Price

Marwan Musa¹, J Massalesse¹ and Diaraya¹

¹ Mathematics Department, Faculty of Mathematics and Natural Sciences, Hasanuddin University, Jl. Perintis Kemerdekaan 10 Tamalanrea Makassar, Sulawesi Selatan, 90245, Indonesia.

E-mail: marwanmusa@gmail.com

Abstract. Options are rights to buy or sell underlying assets for an exercise price (strike price), which is fixed by the terms of the option contract. In this research, option price fluctuations determined by stocks as its underlying asset. In binomial model, stock price fluctuations modeled by assuming that stock price will be in one of two circumstances at maturity date, namely rising or falling. Every possibility of stock value at maturity date can be used to discover every option value (payoff) so that it can be used as a consideration for the holder to execute the option and also for the writer to discover the maximum risk of loss. In this research the option price value V_n will be determined by the value of replication portfolio X_n formed through the hedging process in such a way that $V_n = X_n$ for $1 \leq n \leq N$.

1. Introduction

Option are rights to buy or sell underlying assets for an exercise price (strike price), which is fixed by the terms of the option contract [9]. From the definition, options can be divide into two types, namely call option and put option. A call option is the right, but not the obligation, to buy some asset for a specified price on or before a certain date. A put option is the right, but not the obligation, to sell some asset for a specified price on or before a certain date [6]. Trading of standardized options contracts on a national exchange started in 1973 when the Chicago Board options Exchange (CBOE) began listing call options. These contracts were almost immediately a great success, crowding out the previously existing over-the-counter trading in stock options. Options contract are traded now on several exchanges. They are written on common stock, stock indexes, foreign exchange, agricultural commodities, precious metals, and interest rate futures [2].

Stocks are included in the type of risk assets. It is because, the fluctuations of the stock price in the market are very dynamic. The price of one share at time t will be denoted by $S(t)$. The current stock price $S(0)$ is known to all investors, but the future price $S(t)$ remains uncertain: it may go up as well as down [3]. To minimize risk in investment, one of the solution is to use stocks option contract.

Based on exercise date, option can be divided into two types, namely European option and American option. European option is a contract giving the holder the right to buy or sell an asset, called the underlying, for a specified price fixed in advance, known as the exercise price or strike price, at a

23 specified future time, called the exercise or expiry time. American option gives the right to buy or, respectively, to sell the underlying asset for the strike price at any time between now and a specified future time, called the expiry time. In other words, American option can be exercised at any time up to and including expiry [5].

It has partially developed that the binomial method can be used to approach the option pricing by treating the option pricing problem in its simplest nontrivial setting, the uncertainty of the underlying stock price reduced to discrete binomial [4][10]. Furthermore, by considering a stock whose price can either advance or decline during the next period, it is possible to form a riskless portfolio with the two securities with the assumption that the prices of the stock and its option follow a two-state process [7].

There is a strategy to reduce the investment risk of option trading by replicating the option through the stock and money market, called replicating portfolio strategy, since this replicating portfolio creates return identical to the option [8][11] and construct a portfolio whose wealth at time one is equal to the value of the option, regardless of the stock price fluctuations, it can be inferred that the value of the option at time zero is simply that of the replicating portfolio [12].

2. No-Arbitrage Stock Option

In binomial model of stock price, no arbitrage may exist. Arbitrage is a trading strategy which :

1. Begins with zero money,
2. Has a zero probability of losing money, and
3. Has positive probability of making money.

Proposition 2.1 Suppose u is the factor of stock price rising, d is the factor of stock price decline, and r is the interest rate. If there is no arbitrage then $0 < d < 1 + r < u$ [12].

Proposition 2.2 if $0 < d < 1 + r < u$, then there is no arbitrage [12].

Proposition 2.3 Suppose u is the factor of stock price rising, d is the factor of stock price decline, and r is the interest rate. There is no arbitrage in stock price if and only if $0 < d < 1 + r < u$.

3. Establishment of Portfolio Replication

Let's suppose the price of European call option is V_0 and see what conditions one can put on V_0 . Suppose the investor start out with V_0 . One thing the investor as the holder could do is buy one option. The other thing the investor as the writer could do is use the money to buy Δ_0 shares of stock. If $V_0 > \Delta_0 S_0$, there will be some money left over and the writer put that in the bank. $V_0 < \Delta_0 S_0$, the writer does not have enough money to buy the stock, and the writer makes up the shortfall by borrowing money from the bank, in either case, at this point the writer have $V_0 - \Delta_0 S_0$ in the bank and Δ_0 shares of stock

$$X_0 = \Delta_0 S_0 + (V_0 - \Delta_0 S_0) = V_0 \quad (1)$$

4 at time 1, the value of this portfolio will be X_1 which consist of $\Delta_0 S_1$ in stock and $V_0 - \Delta_0 S_0$ in the bank will grow into $(1 + r)(X_0 - \Delta_0 S_0)$ as a savings or debt.

$$X_1 = \Delta_0 S_1 + (1 + r)(X_0 - \Delta_0 S_0) = (1 + r)X_0 + \Delta_0(S_1 - (1 + r)S_0) \quad (2)$$

if the stock goes up, the writer will have

$$X_1 = \Delta_0 u S_0 + (1 + r)(X_0 - \Delta_0 S_0), \quad (3)$$

and if it goes down,

$$X_1 = \Delta_0 d S_0 + (1 + r)(X_0 - \Delta_0 S_0) \quad (4)$$

[1].

4. One-Period Binomial Model

7 In binomial model, stock price movements modeled with simple mathematics. In this case, stock price movements from the beginning of the period with a price per share of $S_0 > 0$, at the end of period, it can be determined from the result of coin toss, either heads or tails, with probabilities p and $q = 1 - p$

(p and q are not necessarily $\frac{1}{2}$), respectively. At time one, the price per share will be either $S_1(H)$ or $S_1(T)$, with probabilities p and q . Let

$$u = \frac{S_1(H)}{S_0}, d = \frac{S_1(T)}{S_0} \quad (5)$$

It has been assumed before in proposition 2.1 as a condition for the absence of arbitrage, that both d and u are positive, and without loss of generality, $d < u$.

The result of coin toss affects the stock price at the end of the period, so that the option price is also affected by the result of coin toss. Therefore, possible option prices are

$$V_1(H) = \max\{S_1(H) - K, 0\}$$

$$V_1(T) = \max\{S_1(T) - K, 0\}$$

$V_1(H)$ and $V_1(T)$ represent the option price at the end of period 1 which is very dependent on the value V_0 at the beginning of the period. Therefore by determining the right value of V_0 , the writer can limit the risk of loss from stock price fluctuations.

Theorem 4.1 Suppose u is the factor of stock price rising, d is the factor of stock price decline, and r is the interest rate. If V_0 is the price of the European option at the beginning of the period then the price of V_0 for one-period binomial model is

$$V_0 = \frac{1}{(1+r)} [pV_1(H) + qV_1(T)] \quad (6)$$

for $\frac{1+r-d}{u-d} = p, \frac{u-1+r}{u-d} = q$ and $V_1(H), V_1(T)$ are possible payoffs.

Proof : assume that investors have a capital of X_0 and use that capital to buy shares as much as Δ_0 , so that at the end of the period this replication portfolio will be worth

$$X_1 = \Delta_0 S_1 + (1+r)(X_0 - \Delta_0 S_0) \quad (7)$$

which also depends on the result of the coin toss, so that

$$X_1(H) = \Delta_0 S_1(H) + (1+r)(X_0 - \Delta_0 S_0)$$

$$X_1(T) = \Delta_0 S_1(T) + (1+r)(X_0 - \Delta_0 S_0) \quad (8)$$

X_0 and Δ_0 will be determined so that

$$X_1(H) = V_1(H) \text{ dan } X_1(T) = V_1(T). \quad (9)$$

based on (8) and (9), these two equations take the form

$$X_0 + \Delta_0 \left[\frac{1}{(1+r)} S_1(H) - S_0 \right] = \frac{1}{(1+r)} V_1(H) \quad (10)$$

$$X_0 + \Delta_0 \left[\frac{1}{(1+r)} S_1(T) - S_0 \right] = \frac{1}{(1+r)} V_1(T)$$

Subtracting the second from the first equation on (10) gives

$$\Delta_0 = \frac{V_1(H) - V_1(T)}{S_1(H) - S_1(T)} \quad (11)$$

Next, substituting in the first equation of system (10) formula (11) gives

$$X_0 = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_1(H) + \frac{u-(1+r)}{u-d} \cdot V_1(T) \right] \quad (12)$$

by choosing the right value of Δ_0 in equation (11) so that the value X_0 is obtained as in equation (12) then $X_1 = V_1$ is fulfilled. So that, if the writer start with X_0 as in equation (1) and at the beginning of the period buys Δ_0 shares of stock, then at time 1 or at $t = 1$, if the result of coin toss is head (H), the writer will have a portfolio that gives the same values as payoff $V_1(H)$ that must be paid to the holder, and if the coin toss is tail (T), then the portfolio will gives the same value as the payoff $V_1(T)$ that must be paid to the holder. This condition shows that the hedging process has been carried out. So that the European call option value of V_1 at the end of the period (or at $t = 1$) must be

$$V_0 = \frac{1}{(1+r)} [\bar{p}V_1(H) + \bar{q}V_1(T)] \quad (13)$$

at $t = 0$. So the theorem 4.1 is proven.

5. Two-Period Binomial Model

Based on assumptions, it is known that stock prices rise by factor u and decrease by factor d , so that the stock price values at the end of period two are as follows

$$\begin{aligned} S_2(HH) &= uS_1(H) = u^2S_0, S_2(HT) = dS_1(H) = duS_0, \\ S_2(TH) &= uS_1(T) = udS_0, S_2(TT) = dS_1(T) = d^2S_0 \end{aligned} \quad (14)$$

and the possible option prices are

$$\begin{aligned} V_2(HH) &= \max\{S_2(HH) - K, 0\}, V_2(HT) = \max\{S_2(HT) - K, 0\} \\ V_2(TH) &= \max\{S_2(TH) - K, 0\}, V_2(TT) = \max\{S_2(TT) - K, 0\} \end{aligned} \quad (15)$$

At the beginning of period two, the writer have obtained capital as much as X_1 from investment in the previous period. Furthermore, the capital is used to buy shares as much as Δ_1 so that at the end of period two, this portfolio will be worth

$$\begin{aligned} X_2(HH) &= \Delta_1 S_2(HH) + (1+r)[X_1(H) - \Delta_1(H)S_1(H)] \\ X_2(HT) &= \Delta_1(H)S_2(HT) + (1+r)[X_1(H) - \Delta_1(H)S_1(H)] \\ X_2(TH) &= \Delta_1(T)S_2(TH) + (1+r)[X_1(T) - \Delta_1(T)S_1(T)] \\ X_2(TT) &= \Delta_1(T)S_2(TT) + (1+r)[X_1(T) - \Delta_1(T)S_1(T)] \end{aligned} \quad (16)$$

$\Delta_1(H)$, $\Delta_1(T)$, $X_1(H)$, and $X_1(T)$ will be determined so that

$$X_2(HH) = V_2(HH), X_2(HT) = V_2(HT), X_2(TH) = V_2(TH) \text{ dan } X_2(TT) = V_2(TT) \quad (17)$$

based on (16) and (17) these four equation take the form

$$\begin{aligned} X_1(H) + \Delta_1(H) \left[\frac{1}{(1+r)} S_2(HH) - S_1(H) \right] &= \frac{1}{(1+r)} V_2(HH) \\ X_1(H) + \Delta_1(H) \left[\frac{1}{(1+r)} S_2(HT) - S_1(H) \right] &= \frac{1}{(1+r)} V_2(HT) \\ X_1(T) + \Delta_1(T) \left[\frac{1}{(1+r)} S_2(TH) - S_1(T) \right] &= \frac{1}{(1+r)} V_2(TH) \\ X_1(T) + \Delta_1(T) \left[\frac{1}{(1+r)} S_2(TT) - S_1(T) \right] &= \frac{1}{(1+r)} V_2(TT) \end{aligned} \quad (18)$$

Subtracting the second from the first equation on (18) gives

$$\Delta_1(H) = \frac{V_2(HH) - V_2(HT)}{S_2(HH) - S_2(HT)} \quad (19)$$

Subtracting the fourth from the third equation on (18) gives

$$\Delta_1(T) = \frac{V_2(TH) - V_2(TT)}{S_2(TH) - S_2(TT)} \quad (20)$$

Next, substituting in the first equation of system (18) formula (19) gives

$$X_1(H) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_2(HH) + \frac{u-(1+r)}{u-d} \cdot V_2(HT) \right] \quad (21)$$

and substituting in the third equation of system (18) formula (20) gives

$$X_1(T) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_2(TH) + \frac{u-(1+r)}{u-d} \cdot V_2(TT) \right] \quad (22)$$

by providing capital of $X_1(H)$ or $X_1(T)$ at the beginning of period two based on equation in (21) and (22) and using that capital to buy shares as much as $\Delta_1(H)$ or $\Delta_1(T)$, then at the end of period two, if the result of coin toss is head (H), the writer will have a portfolio that gives the same values as payoff $V_2(HH)$ and $V_2(TH)$ that must be paid to the holder, and if the coin toss is tail (T), then the portfolio will give same value as the payoff $V_2(HT)$ and $V_2(TT)$ that must be paid to the holder. So that the European call option value of V_2 at the end of period two (or at $t = 2$) at $t = 1$ must be

$$V_1(H) = \frac{1}{(1+r)} [\bar{p}V_2(HH) + \bar{q}V_2(HT)] \text{ and } V_1(T) = \frac{1}{(1+r)} [\bar{p}V_2(TH) + \bar{q}V_2(TT)] \quad (23)$$

6. Three-Period Binomial Model

The stock price value at the end of period three is also can be determined by the result of three times coin toss namely heads (H) for the situation of the stock price rising with factor u and tails (T) for the condition of the stock price decreasing with factor d , so that the stock price values at the end of period three are

$$\begin{aligned} S_3(HHH) &= uS_2(HH) = u^2S_1(H) = u^3S_0, S_3(HHT) = dS_2(HH) = duS_1(H) = du^2S_0, \\ S_3(HTH) &= uS_2(HT) = udS_1(H) = uduS_0, S_3(HTT) = dS_2(HT) = d^2S_1(H) = d^2uS_0, \\ S_3(THH) &= uS_2(TH) = u^2S_1(T) = u^2dS_0, S_3(THT) = dS_2(TH) = duS_1(T) = dudS_0, \\ S_3(TTH) &= uS_2(TT) = udS_1(T) = ud^2S_0, S_3(TTT) = dS_2(TT) = d^2S_1(T) = d^3S_0 \end{aligned} \quad (24)$$

and the possible option prices at the end of period three are

$$\begin{aligned} V_3(HHH) &= \max\{S_3(HHH) - K, 0\}, V_3(HHT) = \max\{S_3(HHT) - K, 0\}, \\ V_3(HTH) &= \max\{S_3(HTH) - K, 0\}, V_3(HTT) = \max\{S_3(HTT) - K, 0\}, \\ V_3(THH) &= \max\{S_3(THH) - K, 0\}, V_3(THT) = \max\{S_3(THT) - K, 0\}, \\ V_3(TTH) &= \max\{S_3(TTH) - K, 0\}, V_3(TTT) = \max\{S_3(TTT) - K, 0\} \end{aligned} \quad (25)$$

Based on the previous periods, at the beginning of period three, the writer have obtained capital as much as X_2 from the investment in the previous period. Furthermore, the capital is used to buy shares as much as Δ_2 so that at the end of period three, this portfolio will be worth

$$\begin{aligned}
 X_3(HHH) &= \Delta_2(HH)S_3(HHH) + (1+r)[X_2(HH) - \Delta_2(HH)S_2(HH)] \\
 X_3(HHT) &= \Delta_2(HH)S_3(HHT) + (1+r)[X_2(HH) - \Delta_2(HH)S_2(HH)] \\
 X_3(HTH) &= \Delta_2(HT)S_3(HTH) + (1+r)[X_2(HT) - \Delta_2(HT)S_2(HT)] \\
 X_3(HTT) &= \Delta_2(HT)S_3(HTT) + (1+r)[X_2(HT) - \Delta_2(HT)S_2(HT)] \\
 X_3(THH) &= \Delta_2(TH)S_3(THH) + (1+r)[X_2(TH) - \Delta_2(TH)S_2(TH)] \\
 X_3(THT) &= \Delta_2(TH)S_3(THT) + (1+r)[X_2(TH) - \Delta_2(TH)S_2(TH)] \\
 X_3(TTH) &= \Delta_2(TT)S_3(TTH) + (1+r)[X_2(TT) - \Delta_2(TT)S_2(TT)] \\
 X_3(TTT) &= \Delta_2(TT)S_3(TTT) + (1+r)[X_2(TT) - \Delta_2(TT)S_2(TT)]
 \end{aligned} \tag{26}$$

$\Delta_2(HH), \Delta_2(HT), \Delta_2(TH), \Delta_2(TT), X_2(HH), X_2(HT), X_2(TH),$ and $X_2(TT)$ will be determined so that

$$\begin{aligned}
 X_3(HHH) &= V_3(HHH), X_3(HHT) = V_3(HHT), X_3(HTH) = V_3(HTH), X_3(HTT) = V_3(HTT), \\
 X_3(THH) &= V_3(THH), X_3(THT) = V_3(THT), X_3(TTH) = V_3(TTH), X_3(TTT) = V_3(TTT)
 \end{aligned} \tag{27}$$

based on (26) and (27) these eight equation take the form

$$\begin{aligned}
 X_2(HH) + \Delta_2(HH) \left[\frac{1}{(1+r)} S_3(HHH) - S_2(HH) \right] &= \frac{1}{(1+r)} V_3(HHH) \\
 X_2(HH) + \Delta_2(HH) \left[\frac{1}{(1+r)} S_3(HHT) - S_2(HH) \right] &= \frac{1}{(1+r)} V_3(HHT) \\
 X_2(HT) + \Delta_2(HT) \left[\frac{1}{(1+r)} S_3(HTH) - S_2(HT) \right] &= \frac{1}{(1+r)} V_3(HTH) \\
 X_2(HT) + \Delta_2(HT) \left[\frac{1}{(1+r)} S_3(HTT) - S_2(HT) \right] &= \frac{1}{(1+r)} V_3(HTT) \\
 X_2(TH) + \Delta_2(TH) \left[\frac{1}{(1+r)} S_3(THH) - S_2(TH) \right] &= \frac{1}{(1+r)} V_3(THH) \\
 X_2(TH) + \Delta_2(TH) \left[\frac{1}{(1+r)} S_3(THT) - S_2(TH) \right] &= \frac{1}{(1+r)} V_3(THT) \\
 X_2(TT) + \Delta_2(TT) \left[\frac{1}{(1+r)} S_3(TTH) - S_2(TT) \right] &= \frac{1}{(1+r)} V_3(TTH) \\
 X_2(TT) + \Delta_2(TT) \left[\frac{1}{(1+r)} S_3(TTT) - S_2(TT) \right] &= \frac{1}{(1+r)} V_3(TTT)
 \end{aligned} \tag{28}$$

Subtracting the second from the first equation on (28) gives

$$\Delta_2(HH) = \frac{V_3(HHH) - V_3(HHT)}{S_3(HHH) - S_3(HHT)} \tag{29}$$

Subtracting the fourth from the third equation on (28) gives

$$\Delta_2(HT) = \frac{V_3(HTH) - V_3(HTT)}{S_3(HTH) - S_3(HTT)} \tag{30}$$

Subtracting the sixth from the fifth equation on (28) gives

$$\Delta_2(TH) = \frac{V_3(THH) - V_3(THT)}{S_3(THH) - S_3(THT)} \tag{31}$$

Subtracting the eighth from the seventh equation on (28) gives

$$\Delta_2(TT) = \frac{V_3(TTH) - V_3(TTT)}{S_3(TTH) - S_3(TTT)} \tag{32}$$

Next, substituting in the first equation of system (28) formula (29) gives

$$X_2(HH) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_3(HHH) + \frac{u-(1+r)}{u-d} \cdot V_3(HHT) \right], \quad (33)$$

substituting in the third equation of system (28) formula (30) gives

$$X_2(HT) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_3(HTH) + \frac{u-(1+r)}{u-d} \cdot V_3(HTT) \right], \quad (34)$$

substituting in the fifth equation of system (28) formula (31) gives

$$X_2(TH) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_3(THH) + \frac{u-(1+r)}{u-d} \cdot V_3(THT) \right], \quad (35)$$

and substituting in the seventh equation of system (28) formula (32) gives

$$X_2(TT) = \frac{1}{1+r} \left[\frac{(1+r)-d}{u-d} \cdot V_3(TTH) + \frac{u-(1+r)}{u-d} \cdot V_3(TTT) \right]. \quad (36)$$

By providing capital of $X_2(HH)$, $X_2(HT)$, $X_2(TH)$ or $X_2(TT)$ at the beginning of period three based on equation in (33), (36), (37) and (38), and using that capital to buy shares as much as $\Delta_2(HH)$, $\Delta_2(HT)$, $\Delta_2(TH)$ or $\Delta_2(TT)$, then at the end of period three, if the result of coin toss is head (H), the writer will have a portfolio that gives the same values as payoff $V_3(HHH)$, $V_3(HTH)$, $V_3(THH)$ and $V_3(TTH)$ that must be paid to the holder, and if the coin toss is tail (T), then the portfolio will give the same value as the payoff of $V_3(HHT)$, $V_3(HTT)$, $V_3(THT)$ and $V_3(TTT)$ that must be paid to the holder. So that the European call option value of V_3 at the end of period three (or at $t = 3$) at $t = 2$ must be

$$\begin{aligned} V_2(HH) &= \frac{1}{(1+r)} [\bar{p}V_3(HHH) + \bar{q}V_3(HHT)], & V_2(HT) &= \frac{1}{(1+r)} [\bar{p}V_3(HTH) + \bar{q}V_3(HTT)], \\ V_2(TH) &= \frac{1}{(1+r)} [\bar{p}V_3(THH) + \bar{q}V_3(THT)] & \text{and } V_2(TT) &= \frac{1}{(1+r)} [\bar{p}V_3(TTH) + \bar{q}V_3(TTT)] \end{aligned} \quad (37)$$

7. N-Period Binomial Model

Stock price movements at the beginning of the period N with a price per share S_{N-1} and at the end of period the N the price per share will be S_N can be determined from the result of coin toss, either heads (H) or tails (T). Therefore, at the end of period N or at maturity date time N , stock price value based on every possible result of coin toss N times are

$$\begin{aligned} S_N(\omega_1\omega_2 \dots \omega_{N-1}H) &= uS_{N-1}(\omega_1\omega_2 \dots \omega_{N-1}) \\ &\vdots \\ S_N(\omega_1\omega_2 \dots \omega_{N-1}T) &= dS_{N-1}(\omega_1\omega_2 \dots \omega_{N-1}) \end{aligned} \quad (38)$$

stock price movements also affect the option price, then the option price at the end of the period N will be one of this

$$\begin{aligned} V_N(\omega_1\omega_2 \dots \omega_{N-1}H) &= \max\{S_N(\omega_1\omega_2 \dots \omega_{N-1}H) - K, 0\} \\ &\vdots \\ V_N(\omega_1\omega_2 \dots \omega_{N-1}T) &= \max\{S_N(\omega_1\omega_2 \dots \omega_{N-1}T) - K, 0\} \end{aligned} \quad (39)$$

Based on previous periods, it can be concluded that the composition of the replication portfolio as the capital that the writer must have at the end of the period $n + 1$ must be worth

$$X_{n+1} = \Delta_n S_{n+1} + (1 + r)(X_n - \Delta_n S_n) \quad (40)$$

The equation (40) depends on the values of random variables X (the capital / replication portfolio), Δ (number of shares), dan S (stock price per share). Stock price S is determined by the market, while the amount of capital and the number of shares to be purchased to form the right composition of the replication portfolio must be determined by the writer at the beginning of the period, so that at maturity, the capital or the replication portfolio give the same value as the option payoff that must be paid to the holder.

1 *Theorem 7.1* Consider an N-Period binomial model of European call option, with $0 < d < 1 + r < u$, and with

$$\frac{1 + r - d}{u - d} = \tilde{p}, \frac{u - 1 + r}{u - d} = \tilde{q}. \quad (41)$$

5 Let V_N be a random variable (a call option paying off at time N) depending on the first N coin tosses $\omega_1 \omega_2 \dots \omega_N$. Define recursively backward in time the sequence of random variables $V_{N-1}, V_{N-2}, \dots, V_0$ by

$$V_n(\omega_1 \omega_2 \dots \omega_n) = \frac{1}{1 + r} [\tilde{p}V_{n+1}(\omega_1 \omega_2 \dots \omega_n H) + \tilde{q}V_{n+1}(\omega_1 \omega_2 \dots \omega_n T)], \quad (42)$$

17 so that each V_n depends on the first n coin tosses $\omega_1 \omega_2 \dots \omega_n$, where n ranges between $N - 1$ and 0 . Next define

$$\Delta_n(\omega_1 \omega_2 \dots \omega_n) = \frac{V_{n+1}(\omega_1 \omega_2 \dots \omega_n H) - V_{n+1}(\omega_1 \omega_2 \dots \omega_n T)}{S_{n+1}(\omega_1 \omega_2 \dots \omega_n H) - S_{n+1}(\omega_1 \omega_2 \dots \omega_n T)} \quad (43)$$

where again n ranges between 0 and $N - 1$. If we set $X_0 = V_0$ and define recursively forward in time the portfolio values X_1, X_2, \dots, X_N by (40), then we will have

$$X_N(\omega_1 \omega_2 \dots \omega_N) = V_N(\omega_1 \omega_2 \dots \omega_N) \text{ for all } \omega_1 \omega_2 \dots \omega_N. \quad (44)$$

for $n = 1, 2, \dots, N$, the random variable $V_n(\omega_1 \omega_2 \dots \omega_n)$ defined to be the value of the option at time n if the outcomes of the first n tosses are $\omega_1 \omega_2 \dots \omega_n$. The value of the option at time zero is defined to be V_0 [11].

1 *Proof:* We prove by forward induction on n that

$$X_n(\omega_1 \omega_2 \dots \omega_n) = V_n(\omega_1 \omega_2 \dots \omega_n) \text{ for all } \omega_1 \omega_2 \dots \omega_n, \quad (45)$$

where $0 \leq n \leq N$. The case of $n = 0$ is given by the definition of X_0 as V_0 . Next, we will prove $X_n = V_n$ with $0 \leq n \leq N$

- Assume that $X_n = V_n$ holds for $n < N$
- we will show that $X_{n+1} = V_{n+1}$ is also holds.

1 The outcome of the first of $n + 1$ coin tosses are $\omega_1 \omega_2 \dots \omega_{n+1}$ and $\omega_{n+1} = \{H, T\}$, and we don't know whether $\omega_{n+1} = H$ or $\omega_{n+1} = T$, so we consider both cases.

Case I : for $X_{n+1}(\omega_1\omega_2 \dots \omega_n H)$

It has been known that the value of Δ_n in equation (43) can be written simply as follows

$$\Delta_n = \frac{V_{n+1}(H) - V_{n+1}(T)}{(u-d)S_n} \quad (46)$$

substituting in the equation (40) formula (46) gives

$$X_{n+1}(H) = (1+r)X_n + \frac{(V_{n+1}(H) - V_{n+1}(T))S_n(u - (1+r))}{(u-d)S_n} \quad (47)$$

We have assumed that $X_n = V_n$ holds for $n < N$, so that

$$X_{n+1}(H) = (1+r)V_n + \bar{q}V_{n+1}(H) - \bar{q}V_{n+1}(T)$$

Substituting in the equation above formula (42) gives

$$\begin{aligned} X_{n+1}(H) &= \bar{p}V_{n+1}(H) + \bar{q}V_{n+1}(T) + \bar{q}V_{n+1}(H) - \bar{q}V_{n+1}(T) \\ &= \left(\frac{1+r-d}{u-d} + \frac{u-1+r}{u-d} \right) V_{n+1}(H) \\ &= \left(\frac{u-d}{u-d} \right) V_{n+1}(H) \\ &= V_{n+1}(H) \quad \text{if } u-d \neq 0 \end{aligned}$$

$u-d \neq 0$ because $0 < d < 1+r < u$, so that

$$X_{n+1}(\omega_1\omega_2 \dots \omega_n H) = V_{n+1}(\omega_1\omega_2 \dots \omega_n H) \quad (48)$$

Case II : for $X_{n+1}(\omega_1\omega_2 \dots \omega_n T)$

substituting in the equation (40) formula (46) gives

$$X_{n+1}(T) = (1+r)X_n + \frac{(V_{n+1}(H) - V_{n+1}(T))S_n(d - (1+r))}{(u-d)S_n} \quad (49)$$

We have assumed that $X_n = V_n$ holds for $n < N$, so that

$$X_{n+1}(T) = (1+r)V_n - \bar{p}V_{n+1}(H) + \bar{p}V_{n+1}(T)$$

Substituting in the equation above formula (42) gives

$$\begin{aligned} X_{n+1}(T) &= \bar{p}V_{n+1}(H) + \bar{q}V_{n+1}(T) - \bar{p}V_{n+1}(H) + \bar{p}V_{n+1}(T) \\ &= \left(\frac{1+r-d}{u-d} + \frac{u-1+r}{u-d} \right) V_{n+1}(T) \\ &= \left(\frac{u-d}{u-d} \right) V_{n+1}(T) \\ &= V_{n+1}(T) \quad \text{if } u-d \neq 0 \end{aligned}$$

$u-d \neq 0$ because $0 < d < 1+r < u$, so that

$$X_{n+1}(\omega_1\omega_2 \dots \omega_n T) = V_{n+1}(\omega_1\omega_2 \dots \omega_n T) \quad (50)$$

Hence, the equations (48) and (50) show that for $\omega_{n+1} = H$ or $\omega_{n+1} = T$, will give the result

$$X_{n+1}(\omega_1\omega_2 \dots \omega_n\omega_{n+1}) = V_{n+1}(\omega_1\omega_2 \dots \omega_n\omega_{n+1}) \quad (51)$$

holds for all $\omega_1\omega_2 \dots \omega_n\omega_{n+1}$.

8. Conclusion

Option price model for n -period is

$$V_n(\omega_1\omega_2 \dots \omega_n) = \frac{1}{1+r} [\bar{p}V_{n+1}(\omega_1\omega_2 \dots \omega_n H) + \bar{q}V_{n+1}(\omega_1\omega_2 \dots \omega_n T)] \quad (52)$$

where V_n a random variable (a call option paying off at time N) depending on the first n coin tosses $\omega_1\omega_2 \dots \omega_n$ with $0 \leq n \leq N - 1$. V_n also defined as the no-arbitrage option price at $t = n$ with a condition $0 < d < 1 + r < u$. To limit the risk of very dynamic stock price fluctuations, then the writer must form a replication portfolio with the right composition, by purchase Δ_n shares of stock

$$\Delta_n(\omega_1\omega_2 \dots \omega_n) = \frac{V_{n+1}(\omega_1\omega_2 \dots \omega_n H) - V_{n+1}(\omega_1\omega_2 \dots \omega_n T)}{S_{n+1}(\omega_1\omega_2 \dots \omega_n H) - S_{n+1}(\omega_1\omega_2 \dots \omega_n T)} \quad (53)$$

with $0 \leq n \leq N - 1$ so that at the end of the period, the portfolio will give the same value as the payoff that must be paid to the holder

$$X_N(\omega_1\omega_2 \dots \omega_n\omega_N) = V_N(\omega_1\omega_2 \dots \omega_n\omega_N) \text{ for all } \omega_1\omega_2 \dots \omega_n\omega_N \quad (54)$$

with X_1, X_2, \dots, X_N defined recursively forward by

$$X_{n+1} = \Delta_n S_{n+1} + (1+r)(X_n - \Delta_n S_n). \quad (55)$$

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