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## Flattening Property and the Existence of Global Attractors in Banach Space

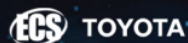
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## Flattening Property and the Existence of Global Attractors in Banach Space

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**Abstract.** This paper analyses the existence of global attractor in infinite dimensional system using flattening property. The earlier stage we show the existence of the global attractor in complete metric space by using concept of the  $\omega$ -limit compact concept with measure of non-compactness methods. Then we show that the  $\omega$ -limit compact concept is equivalent with the flattening property in Banach space. If we can prove there exist an absorbing set in the system and the flattening property holds, then the global attractor exist in the system.

### 1. Introduction

Today many of the physical problems in infinite dimensional system can be represented in mathematical models studied in a strongly continuous semigroup,  $C^0$  semigroups, i.e. the operator family  $\{S_t\}$  that maps a metric space  $M$  to itself and satisfies the properties [1, 2]:

- 1).  $S_{(0)}$  is the identity map on  $M$ ,
- 2).  $S_{(t+s)} = S_t \circ S_s$  for all  $t, s \geq 0$ ,
- 3). the function

$$S: [0, \infty) \times M \rightarrow M \\ (t, x) \mapsto S_t(x) \in M$$

continuous at each point  $(t, x) \in [0, \infty) \times M$ .

The long time dynamics of an infinite dimensional system can be described by the global attractor. Global attractors is the set of connected, compact, and invariant set which attracts all bounded set [2, 3]. Normally, the existence of the global attractor can be proved by showing that:

- 1). the absorbing set exist in the system, and
- 2). the semigroup of the system is compact uniform.

Unfortunately, it is very difficult to show that the semigroup is uniformly compact in many problems. Thus in [3], it is given conditions which can easily proven by using the measure of non-compactness (MNC) called  $\omega$ -limit compact. The existence of the global attractor for a  $C^0$  semigroup can be proved if and only if:

- 1). the absorbing set exist in the system, and
- 2). the semigroup is  $\omega$ -limit compact.



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The use of the MNC concept as well, is used by [3] to show the  $\omega$ -limit compact of a  $C^0$  semigroup in a convex Banach space  $X$ . It is shown to be equivalent with the condition known as flattening property. In [4], flattening property is defined as:

Let  $S_t: X \rightarrow X$  be a semigroup, and  $B$  be any bounded set of  $X$ . Then for any  $\varepsilon > 0$ , there exist  $t(B) > 0$  and a finite dimensional subspace  $X_1$  of  $X$ , such that  $(P)(\bigcup_{t \geq t_B} S_t(B))$  is bounded in  $X$  and

$$(I - P)\left(\bigcup_{t \geq t_B} S_t(B)\right) \subset B(0, \varepsilon) \quad \text{for } t \geq t(B),$$

where  $P: X \rightarrow X_1$  is a bounded projection, and  $I: X \rightarrow X$ .

Furthermore, from the paper [2] the authors provide that the existence of a global attractors can be proved by showing that

- 1). the absorbing set exist in the system, and
- 2). the flattening property holds.

From that explanation, we are interested to re-examine the concept in detail to show the existence of a global attractor by using the measure of non-compactness property. The assessment was carried out in this paper.

## 2. Measure of Non-compactness (MNC)

Some concepts of measure of non-compactness and its basic property will be given in this section, (see [4]).

**Definition 2.1** Let  $M$  be a complete metric space and  $B$  be a bounded subset of  $M$ . The measure of non-compactness  $\gamma(B)$  of  $B$  is defined by

$$\gamma(B) = \inf \{ \delta > 0 \mid B \text{ admits a finite cover by sets of diameter } \leq \delta \}.$$

**Lemma 2.1** Let  $M$  be a complete metric space and  $B$  be a bounded subset of  $M$ . A mapping  $\gamma: B \rightarrow [0, \infty)$  is called measure of noncompactness of  $M$  if it satisfies the following property i.e. for every  $Q, Q_1, Q_2 \in B$

- 1).  $\gamma(Q) = 0$  if and only if  $\bar{Q}$  compact
- 2).  $\gamma(\bar{Q}) = \gamma(Q)$
- 3).  $\gamma(Q_1 \cup Q_2) = \max \{ \gamma(Q_1), \gamma(Q_2) \}$
- 4).  $\gamma(Q_1 + Q_2) \leq \gamma(Q_1) + \gamma(Q_2)$
- 5).  $\gamma(Q_1) \leq \gamma(Q_2)$ , whenever  $Q_1 \subset Q_2$ .

**Lemma 2.2** Let  $X$  be a Banach space and  $\gamma$  be the measure of non-compactness,  $B(0, \varepsilon)$  is a open ball of radius  $\varepsilon$  then  $\gamma(B(0, \varepsilon)) = 2\varepsilon$ .

**Lemma 2.3** Let  $M$  be a complete metric space and  $\gamma$  be the measure of non-compactness. Assume that  $(F_n)$  is a sequence of bounded and closed subset of  $M$ , and satisfying

- 1).  $F_n \neq \emptyset$
- 2).  $F_{n+1} \subset F_n, n = 1, 2, \dots$
- 3).  $\gamma(F_n) \rightarrow 0$ , as  $n \rightarrow \infty$

Then  $F = \bigcap_{n=1}^{\infty} F_n$  is nonempty compact set.

**Definition 2.2** Let  $X$  be a convex Banach space. For any bounded set  $B$  of  $X$ ,  $S_t: X \rightarrow X$  and for any  $\varepsilon > 0$ , there exists  $t(B) > 0$  and a finite dimensional subspace  $X_1$  of  $X$ , such that  $(P)(\bigcup_{t \geq t_B} S_t(B))$  is bounded in  $X$  and

$$(I - P) \left( \bigcup_{t \geq t_B} S_t(B) \right) \subset B(0, \varepsilon) \quad \text{for } t \geq t(B),$$

where  $P: X \rightarrow X_1$  is a bounded projection, and  $I: X \rightarrow X$ .

### 3. The Concept of Global Attractor

In this section, we will provide some definitions from global attractors in dynamic systems related to our paper, ([1, 4, 5])

**Definition 3.1** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a complete metric space  $M$ . A subset  $B_0$  of  $M$  is called an absorbing set in  $M$  if for any bounded subset  $B$  of  $M$ , there exists some  $t_1 \geq 0$  such that  $S_t B \subset B_0$  for all  $t \geq t_1$ .

**Definition 3.2** For any subset  $W_1, W_2$  in complete metric space  $M$ , the subset  $W_1$  is called absorbs  $W_2$  under  $\{S_t\}$  if there exists a number  $t_0 \geq 0$  such that  $S_t W_2 \subset W_1$ , for all  $t \geq t_0$ .

Notice that  $W_1$  attracts  $W_2$  if and only if each open neighborhood  $\mathcal{N}_{W_1}$  of  $W_1$  in  $M$  absorbs  $W_2$ .

**Definition 3.3** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a complete metric space  $M$ , the semigroup is called  $\omega$ -limit compact if for every bounded subset  $B$  of  $M$  and for any  $\varepsilon > 0$ , there exists a  $t_0 > 0$  such that

$$\gamma \left( \bigcup_{t \geq t_0} S_t(B) \right) \leq \varepsilon.$$

**Definition 3.4** Let  $M$  be a metric space. A subset  $\mathcal{A} \subset M$  is called global attractor if  $\mathcal{A}$  is attractor compact that attracts all bounded sets of  $M$ .

**Lemma 3.1** Let  $\bigcup_{t \geq t_B} S_t(B)$  is bounded, then  $\overline{\bigcup_{t \geq t_B} S_t(B)}$  is bounded.

Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a metric space  $M$ . If  $S_t$  is  $\omega$ -limit compact, then  $\mathcal{A} = \omega(B)$  is compact set.

**Lemma 3.2** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a metric space  $M$ . Then  $\mathcal{A} = \omega(B)$  is an invariant positive, i.e.  $S_t \mathcal{A} \subset \mathcal{A}$ .

**Lemma 3.3** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a metric space  $M$ , and  $\mathcal{A} = \omega(B)$  is compact set. Then  $\mathcal{A}$  is an invariant negative, i.e.  $S_t \mathcal{A} \supset \mathcal{A}$ .

**Lemma 3.4** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a metric space  $M$ . If there exists a bounded absorbing subset  $B$  of  $M$ , then  $\mathcal{A} = \omega(B)$  attracts  $B$  of  $M$ .

### 4. Main Result

In this section, firstly we recall some basic lemmas in [3, 5], secondly we show the existence of global attractor by concept  $\omega$ -limit compact with measure of noncompactness methods in a metric space complete. Then we show that concept  $\omega$ -limit compact equivalent with the flattening property in Banach space.

**Theorem 4.1** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a complete metric space  $M$ . Assume that

- 1).  $\mathcal{A}$  is  $\omega$ -limit compact,
- 2). there exists a bounded absorbing subset  $B$  of  $M$ .

Then the  $\omega$ -limit set of  $B$ ,  $\mathcal{A} = \omega(B)$  is compact attractor which attracts all bounded subsets of  $M$ .

**Proof:**

In proving that  $\mathcal{A} = \omega(B)$  is compact attractor, firstly we show that  $\mathcal{A}$  is compact and  $\mathcal{A}$  attractor. From Lemma 3.2, we prove that  $\mathcal{A}$  is compact. To show that  $\mathcal{A}$  attractor, as well as showing  $\mathcal{A}$  invariant set and  $\mathcal{A}$  attracts  $B$  of  $M$ .  $\mathcal{A}$  invariant set, it is shown that  $\mathcal{A}$  is invariant positive and  $\mathcal{A}$  is invariant negative. From Lemma 3.3 and Lemma 3.4, it can be concluded that  $\mathcal{A}$  is invariant.

Furthermore, we will show that  $\mathcal{A}$  attract  $B$  of  $M$ .

Since there exists a bounded subset  $B$  of  $M$ . From Definition 3.2, we get that  $\mathcal{A}$  attracts  $B$  of  $M$ . Besides that, we can show that  $\mathcal{A}$  attracts  $B$  of  $M$  by using contradictions like Lemma 3.4. Thus,  $\mathcal{A} = \omega(B)$  is a compact attractor that attracts all bounded subsets in  $M$ .

**Theorem 4.2** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a complete metric space  $M$ . Then  $S_t$  has a global attractor  $\mathcal{A}$  in  $M$  if and only if

- 1).  $\{S_t\}_{t \geq 0}$  is  $\omega$ -limit compact, and
- 2). there is a bounded absorbing set  $B \subset M$ .

**Proof:**

( $\Leftarrow$ ) From Theorem 4.1 we have proved that  $\mathcal{A}$  is a compact attractor. Referring to Definition 3.4,  $\mathcal{A}$  is a global attractor.

( $\Rightarrow$ ) Now we will prove the opposite. Since  $\mathcal{A}$  is a global attractor, then  $\mathcal{A}$  attractor and  $\mathcal{A}$  compact (see Definition 3.4). So that the neighborhood  $\mathcal{N}_\varepsilon(\mathcal{A})$  is absorbing set. According to the Definition 3.4  $\mathcal{N}_\varepsilon(\mathcal{A})$  absorbs all bounded sets  $B$  of  $M$ . Next we will prove that the  $\omega$ -limit compact.

Since  $B$  is bounded set, then  $\bigcup_{t \geq t_\varepsilon B} S_t(B)$  is bounded for  $t_\varepsilon(B) \geq 0$ , and for any  $t \geq t_\varepsilon(B)$ .

Since  $\mathcal{N}_\varepsilon(\mathcal{A})$  is the absorbing set then

$$\bigcup_{t \geq t_\varepsilon B} S_t(B) \subset \mathcal{N}_\varepsilon(\mathcal{A}).$$

For  $\varepsilon > 0$  take  $\frac{\varepsilon}{4} > 0$ , such that

$$\bigcup_{t \geq t_\varepsilon B} S_t(B) \subset \mathcal{N}_{\frac{\varepsilon}{4}}(\mathcal{A}).$$

From Lemma 2.2, we have

$$\gamma\left(\bigcup_{t \geq t_\varepsilon B} S_t(B)\right) \leq \gamma\left(\mathcal{N}_{\frac{\varepsilon}{4}}(\mathcal{A})\right).$$

To show that the semigroup is compact, then it is the same as we show that

$$\gamma\left(\bigcup_{t \geq t_\varepsilon B} S_t(B)\right) \leq \gamma\left(\mathcal{N}_{\frac{\varepsilon}{4}}(\mathcal{A})\right) \leq \varepsilon.$$

Since  $\mathcal{A}$  is compact, there exists a finite number of elements  $x_1, x_2, \dots, x_n \in M$  such that

$$\mathcal{A} \subset \bigcup_{i=1}^n \mathcal{N}\left(x_i, \frac{\varepsilon}{4}\right).$$

On the other hand, for every  $y_j \in \mathcal{N}_{\varepsilon_1}(\mathcal{A}), j = 1, 2, \dots, k$  then  $y_j \in \bigcup_{i=1}^n \mathcal{N}(x_i, \varepsilon_2), i = 1, 2, \dots, n$  with  $y_j, x_i \in M$ , and  $\varepsilon_2 \geq \varepsilon_1$ . So that

$$\mathcal{N}_{\frac{\varepsilon}{4}}(\mathcal{A}) \subset \bigcup_{i=1}^n \mathcal{N}\left(x_i, \frac{\varepsilon}{2}\right).$$

Which implies that

$$\gamma\left(\bigcup_{t \geq t_\varepsilon B} S_t(B)\right) \leq \gamma\left(\mathcal{N}_{\frac{\varepsilon}{4}}(\mathcal{A})\right) \leq \varepsilon.$$

Therefore,  $\gamma(\bigcup_{t \geq t_\varepsilon B} S_t(B)) \leq \varepsilon$  has a semigroup which is  $\omega$ -limit compact.

**Theorem 4.3** *Let  $X$  be a Banach space and  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in  $X$ . Then  $\{S_t\}_{t \geq 0}$  is  $\omega$ -limit compact if and only if flattening property holds true.*

*Proof.* ( $\Leftarrow$ ) From Definition 3.3, we know that to show  $\{S_t\}_{t \geq 0}$  is  $\omega$ -limit compact equal to indicating that for any bounded absorbing sets  $B$  in  $X$ , and for every  $\varepsilon > 0$  there exists  $t_B > 0$  such that

$$\gamma\left(\bigcup_{t \geq t_B} S_t(B)\right) \leq \varepsilon.$$

Note that

$$\begin{aligned} \gamma\left(\bigcup_{t \geq t_B} S_t(B)\right) &= \gamma\left((P + I - P)\left(\bigcup_{t \geq t_B} S_t(B)\right)\right) \\ &= \gamma\left(P\left(\bigcup_{t \geq t_B} S_t(B)\right) + (I - P)\left(\bigcup_{t \geq t_B} S_t(B)\right)\right). \end{aligned} \tag{1}$$

From Definition 2.2,  $(P)(\bigcup_{t \geq t_B} S_t(B))$  is bounded, and we get  $(P)(\overline{\bigcup_{t \geq t_B} S_t(B)})$  bounded (see Lemma 3.1).

From closure definition [6], we have

$$\overline{\bigcup_{t \geq t_B} S_t(B)} = \bigcup_{t \geq t_B} S_t(B) \cup \left(\bigcup_{t \geq t_B} S_t(B)\right)'$$

In other word,

$$\left(\bigcup_{t \geq t_B} S_t(B)\right)' \subset \overline{\bigcup_{t \geq t_B} S_t(B)}.$$

From the theorem of the close set on [6], it is found that  $\overline{\bigcup_{t \geq t_B} S_t(B)}$  closed and  $(p)(\overline{\bigcup_{t \geq t_B} S_t(B)})$  also closed. So we can conclude that  $(p)(\overline{\bigcup_{t \geq t_B} S_t(B)})$  is compact.

From Lemma 2.2 (1), we have

$$\gamma\left(p\left(\overline{\bigcup_{t \geq t_B} S_t(B)}\right)\right) = 0.$$

From the conclusion of Lemma 2.2, Definition 2.2, and (1), we have

$$\gamma\left(\bigcup_{t \geq t_B} S_t(B)\right) = \gamma\left(P\left(\bigcup_{t \geq t_B} S_t(B)\right) + (I - P)\left(\bigcup_{t \geq t_B} S_t(B)\right)\right)$$

$$\begin{aligned} &\leq \gamma \left( P \left( \bigcup_{t \geq t_B} S_t(B) \right) \right) + \gamma \left( (I - P) \left( \bigcup_{t \geq t_B} S_t(B) \right) \right) \\ &\leq 0 + \gamma(B(0, \varepsilon)) = 2\varepsilon. \end{aligned}$$

Therefore,  $S_t$  is  $\omega$ -limit compact.

( $\Rightarrow$ ) Since  $S_t$  is  $\omega$ -limit compact, then for any bounded absorbing sets  $B$  in  $X$ , and for every  $\varepsilon > 0$  there exists  $t_B > 0$  such that

$$\gamma \left( \bigcup_{t \geq t_B} S_t(B) \right) \leq \frac{\varepsilon}{2}.$$

Let  $A_1, A_2, \dots, A_n$  be subset of  $A$  with diameter less than  $\frac{\varepsilon}{2}$ , such that

$$\bigcup_{t \geq t_B} S_t(B) \subset \bigcup_{i=1}^n A_i.$$

Let  $x_i \in A_i$ , and diameter less than  $\varepsilon$  then

$$\bigcup_{t \geq t_B} S_t(B) \subset \bigcup_{i=1}^n \mathcal{N}(x_i, \varepsilon).$$

Since  $X$  is uniformly convex Banach space, and let  $X_1 = \text{span}\{x_1, \dots, x_n\}$ , there exists a projection  $P: X \rightarrow X_1$  such that  $\text{dist}(x, Px) = \|(I - P)x\| = \text{dist}(x, X_1)$ , for any  $x \in X$ . Hence,

$$(I - P) \left( \bigcup_{t \geq t_B} S_t(B) \right) \subset \mathcal{N}(0, \varepsilon). \tag{2}$$

Since  $B$  is bounded, then  $\bigcup_{t \geq t_B} S_t(B)$  is bounded in  $X$  (see theorem of close set on [6]). Hence the following equation is bound in  $X$ .

$$(P) \left( \bigcup_{t \geq t_B} S_t(B) \right). \tag{3}$$

From (2) and (3), Definition 2.2 is met. Namely flattening property holds true. ■

**Theorem 4.4** Let  $\{S_t\}_{t \geq 0}$  be a  $C^0$  semigroup in a Banach space  $X$ . If the following conditions

- 1). There exist a bounded absorbing set  $B \subset X$ , and
- 2).  $\{S_t\}_{t \geq 0}$  satisfies the flattening property

hold true, then there is a global attractor for  $\{S_t\}_{t \geq 0}$  in  $X$ .

**Proof:**

Using Theorem 4.3, it has been shown that  $\{S_t\}_{t \geq 0}$  meets the flattening property. In addition, referring to Theorem 4.2 we can conclude that there is a bounded absorbing set  $B \subset X$ . Therefore the conditions in Theorem 4.4 have been met. The existence of a global attractor for  $\{S_t\}_{t \geq 0}$  in  $X$  is guaranteed to be true.

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