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Harmonious labeling on prisms graph

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Abstract. In this paper, we show that $C_{2a+1} \times P_n$ and $(C_{2a+1} \times P_n) + \overline{K_p}$ is harmonious for $a \geq 1$, and n is even.

1. Introduction

In general, a graph is a diagram that contains information that is interpreted appropriately. In everyday life, graphs are used for various views of existing structures. The aim is to visualize the relationship between objects to make them easier to understand. Some examples of graphs that are often found in everyday life include organizational structures, course charts, maps, electrical circuits, etc

A graph G is a set of pairs (V, E) written with notation G = (V, E), with V is a non-empty and finite set of objects called vertices and E is a set (maybe empty) pair not sorted from different points in V which are called edges

A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to the positive or non-negative integers). The most common choices of d

omain are the set of all vertices and edges (such labelings are called *total* labelings), the vertex-set alone (*vertez*-labelings), or the edge-set alone (*edge*-labelings). Other domains are possible.

A mapping $\lambda : V \to Z_{|E|}$ where $Z_{|E|}$ is the set of modulo integers $|E|$ called harmonious labeling if λ is a map so that when each edge of xy is labeled $w(xy) = \lambda(x) + \lambda(y)(mod\ |E|)$ produces a label on a different edge, in this case $w(xy) \in \{0,1,\dots,|E|-1\}$. A graph that has harmonious labeling is called a Harmonious Graph.

Based on the writings of Joseph A. Gallian (2018) with the title A Dynamic Survey of Graph Labeling, studies on harmonious labeling on various types of graphs have been conducted. One interesting harmonious labeling according to the author is harmonious labeling on the prism graph $C_m \times P_n$. Previous researchers have proven that the graph $C_m \times P_n$ is a harmonious graph in various cases, including Graham and Sloane (1980) proving that $C_m \times P_n$ is a harmonious graph for n odd numbers, then Gallian A. Joseph, Prout J., and Winters S. (1992) proved that $C_m \times P_n$ is a harmonious graph for n = 2 and m $\neq$ 4. Furthermore D. Jungreis and M. Reid (1992) prove that $C_m \times P_n$ is a harmonious graph for m = 4 and n $\geq$ 3. By referring to the previous research, the author intends to examine the harmonious labeling of the prism graph $C_m \times P_n$ with the condition m is an odd number.

2. Main Result of Harmonious Labeling on Prisms Graph

Before labeling the prism graph, a theorem from the previous researcher will given about harmonious labeling on a ladder graph follows

Theorem 2.1 *Ladder graph L_n for $n > 2$ is a harmonious graph*

L_{2a+1} $(a \geq 1)$ is harmonious: label one path 0, a+1, 1, a+2, 2, a+3,…,a-1, 2a, a, and the other 3a+1, 2a+1, 3a+2, 2a+2, …,4a,3a,4a+1 (Figure 2.1). L_4 is harmonious: label the paths 0, 5, 1, 9 and 2, 6, 3, 4 (Figure 2.2). Finally Figure 2.3 shows a harmonious labeling of L_{2a} for $a \geq 3$. ∎

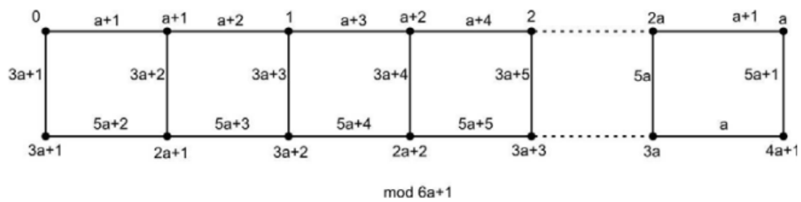

Figure 2.1 L_{2a+1} $(a \geq 1)$

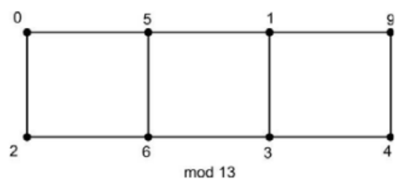

Figure 2.2 L_4 $(a \geq 1)$

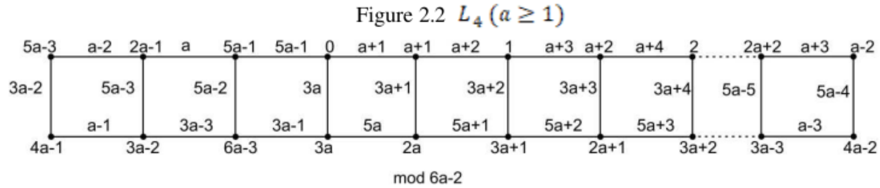

Figure 2.2 L_{2a} $(a \geq 3)$

Graham and Sloane (1980) write that by connecting the starting points and end points on the date graph L_{2a+1} be a harmonious labeling on the graph $C_{2a+1} \times P_2$, and with the same labeling pattern can be proceed to a harmonious labeling on the prism graph $C_{2a+1} \times P_n$.

Theorem 2.1 *Prism graph $C_{2a+1} \times P_n$ is harmonious for $a \geq 1$, n is even*

Proof.

Let

$$A = \{0,1,\dots,a-1,a\} \cup \{2a+1,2a+2,\dots,3a,3a+1\} \cup \{4a+2,4a+3,\dots,5a+2\} \cup \dots \cup \{2(n-1)a+(n-1),2(n-1)a+(n-1)+1,\dots,(2n-1)a+(n-1)\}$$

$$B = \{2a+1,2a+2,\dots,3a,3a+1\} \cup \{6a+3,6a+4,\dots,7a+3\} \cup \{8a+4,8a+5,\dots,9a+4\} \cup \dots \cup \{2(n-1)a+(n-1),2(n-1)a+(n-1)+1,\dots,(2n-1)a+(n-1)\}$$

Consider $C_{2a+1} \times P_n$ with the vertex set $\{v_0, v_1, v_2, \ldots, v_{2na+n-1}\}$. Let G be the graph $C_{2a+1} \times P_n$. Then $|V(G)| = 2na + n$ and $|E(G)| = q = 4na + 2n - 2a - 1$.

Let

$E_i = v_i v_{i+a}$, $i \in A$

$E_j = v_j v_{j+a+1}$, $j \in A - \{a, 3a+1, 5a+2, 7a+3, 9a+4, \ldots\}$

$E_k = v_k v_{k+3a+1}$, $k \in A - B$,

$E_l = v_l v_{l+3a+2}$,

$k \in B - \{2(n-1)a + (n-1), 2(n-1)a + (n-1) + 1, \ldots, (2n-1)a + (n-1)\}$

,

$E_r = v_r v_{r-a-1}$, $r \in A - (B \cup \{0,1, \ldots, a-1, a\}$.

$E_s = v_s v_{s-a}$, $s \in B$.

Then the edge set of G is

$E(G) = \{E_i, E_j, E_k, E_l, E_r, E_s \,|\, i \in A, j \in A - \{a, 3a+1, 5a+2, 7a+3, 9a+4, \ldots\}, k \in A - B, l \in B - \{2(n-1)a + (n-1), 2(n-1)a + (n-1) + 1, \ldots, (2n-1)a + (n-1)\}, r \in A - (B \cup \{0,1, \ldots, a-1, a\}, s \in B\}$

.

Next define the vertex labeling and edge labelling of G as follows

$f(v_x) = x$, for every $x \in Z_{2na+n}$, dan

$g(v_x v_y) = f(v_x) + f(v_y) \mod q$, for every $x, y \in Z_{2na+n}$.

It is clear that edge set labeling are distinct. Hence, the graph G is harmonious graph. ∎

For illustration in Figure 2.1 a labeling is shown on the grid graph $P_{2a+1} \times P_n$. If each endpoint (i.e $a, 4a+1, \ldots$) and base point (i.e. $0, 3a+1, \ldots$) on a horizontal path are connected, it will form a prism graph $C_{2a+1} \times P_n$.

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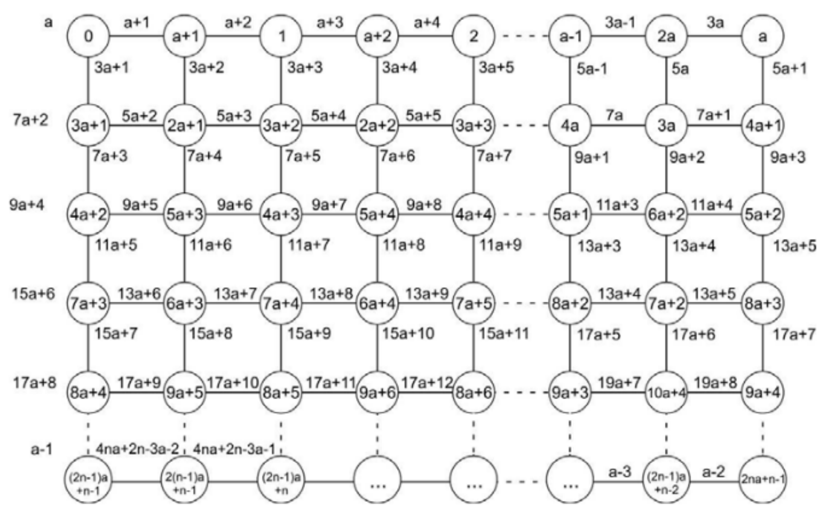

Figure 2.1 $P_{2a+1} \times P_n$

From the labeling construction shown in Figure 2.1 it can be seen that the set of edge graph labels is $g\,(e)\; = \{a, a\; +\; 1, \ldots, 4na\; +\; 2n - 2a - 2,\} \cup \{0,1,2, \ldots, a - 1\}$ where $e \in E(G)$ and it can be seen that each edge label is different so it is proven that harmonious labeling can be done on a prism graph $C_{2a+1} \times P_n$.

Theorem 2.3 *Prisms graph* $C_m \times P_n$ *is harmonious for m is odd and* n *is even*

Proof. Consider $C_m \times P_n$ with the vertex set $\{v_0, v_1, v_2, \ldots, v_{nm-1}\}$. Let G be the graph $C_m \times P_n$. Then $|V(G)| = nm$ and $|E(G)| = q = 2nm - m$

Let

$a_i = v_i v_{i-1}\,, i \in Z_{nm} - \{0, m, 2m, \ldots, (n-1)m\}$,

$b_j = v_j v_{j+m-1}, 0 \leq j \leq (n-1)m$,

$c_k = v_k v_{k+2m-1}, k = 0, m, 2m, \ldots, (n-2)m$,

Then edge set of G is

$E(G) = \{a_i, b_j, c_k \mid i \in Z_{nm} - \{0, m, 2m, \ldots, (n-1)m\}, 0 \leq j \leq (n-1)m, k = 0, m, 2m, \ldots,$
$(n-2)m\}$.

We define the vertex labeling and edge labelling on G as follows

$f(v_x) = x$, for every $x \in Z_{nm}$, and

$g(v_x v_y) = f(v_x) + f(v_y) \bmod q$, for every $x, y \in Z_{nm}$.

for $i \in Z_{nm} - \{0, m, 2m, \ldots, (n-1)m\}$, then

$g(a_i)\; =\; g(v_i v_{i-1})$

$$= f(v_i) + f(v_{i-1}) \ mod\ q$$
$$= (2i - 1) \ mod\ q,$$

for $0 \le j \le (n-1)m$, then

$$g(b_j) = g(v_j v_{j+m-1})$$
$$= f(v_j) + f(v_{j+m-1}) \ mod\ q$$
$$= (2j + m - 1) \ mod\ q,$$

for $k = 0, m, 2m, \dots, (n-2)m$, then

$$g(c_k) = g(v_k v_{k+2m-1})$$
$$= f(v_k) + f(v_{k+2m-1}) \ mod\ q$$
$$= (2k + 2m - 1) \ mod\ q.$$

It is clear that edge set labeling are distinct. Hence, the graph G is harmonious graph. ∎

Theorem 2.2 The graph $(C_{2a+1} \times P_n) + \overline{K_p}$ is harmonious graph for $a \ge 1$ and n is even.

Proof. Consider $(C_{2a+1} \times P_n) + \overline{K_p}$ with the vertex set $V(G) = \{v_0, v_1, v_2, \dots, v_{2na+n-1}\} \cup \{v_{4na+2n-a-1}, v_{(4na+2n-a-1)+(2na+n)}, \dots, v_{(4na+2n-a-1)+(p-1)(2na+n)}\}$ and the edge set $E(G) = E(C_{2a+1} \times P_n) \cup \{v_i v_j | i = (4na + 2n - a - 1), (4na + 2n - a - 1) + (2na + n), \dots, (4na + 2n - a - 1) + (p - 1)(2na + n)\ and\ j = 0,1, \dots, 2na + n - 1\}$. Let G be the graph $(C_{2a+1} \times P_n) + \overline{K_p}$. Then $|V(G)| = 2na + n + p$ and $|E(G)| = q = (4na + 2n - 2a - 1) + p(2na + n)$.

We define the vertex labeling and edge labeling

$f: V \to Z_q$ and $g: E \to Z_q$ where

$$f(v_x) = x \text{, for every } v_x \in V(G)$$

and

$$g(v_x v_y) = f(v_x) + f(v_y) \ mod\ q \text{, for every } v_x, v_y \in V(G)$$

To show that $(C_{2a+1} \times P_n) + \overline{K_p}$ is a harmonious graph, it is sufficient to show that g is a 1-1 map. In Theorem 2.1, a harmonious labeling on the prism graph $C_{2a+1} \times P_n$ has been shown, meaning that each side label on the prism graph is different. So to prove that the graph $(C_{2a+1} \times P_n) + \overline{K_p}$ is harmonized, it is enough to prove that $g(v_i v_j)$ is different for $i = (4na + 2n - a - 1), (4na + 2n - a - 1) + (2na + n), \dots, (4na + 2n - a - 1) + (p - 1)(2na + n)\ and\ j = 0,1,2 \dots, 2na + n - 1\}$

For $i = (4na + 2n - a - 1), (4na + 2n - a - 1) + (2na + n), \dots, (4na + 2n - a - 1) + (p - 1)(2na + n)$ $and\ j = 0,1,2 \dots, 2na + n - 1\}$, we have

Journal of Physics: Conference Series **1341** (2019) 062012 doi:10.1088/1742-6596/1341/6/062012

$g(v_i v_j) = f(v_i) + f(v_j) \ mod \ q$

$= (i + j) mod \ q,$

so that

$$g(v_i v_j) = \begin{cases} (i+j) - q\,, & i = (4na + 2n - a - 1) + (p-1)(2na + n) \ and \\ & j \geq 2na + n - a - 1 \\ i + j, & else \end{cases}$$

1. We will shown that $g(v_i v_j) = i + j \neq i' + j' = g(v_{i'} v_{j'})$ for $i > i'$
   $i - i' \geq 2na + n,$

   because the maximum value of j is $2na + n - 1$

   then $i' + j' < i$. if $i' + j' < i$ then $i' + j' < i + j$.

   So that $g(v_{i'} v_{j'}) = i' + j' < i + j = g(v_i v_j)$ then $g(v_i v_j) \neq g(v_{i'} v_{j'})$.

2. We will shown that $g(v_i v_j) \neq g(v_{i'} v_{j'})$ for

   $i = (4na + 2n - a - 1) + (p-1)(2na + n)$ and

   $j \geq 2na + n - a - 1,$

   $i' \neq i$ or $j' \neq j$.

   the maximum value of $g(v_i v_j)$ will be obtained if $i = (4na + 2n - a - 1) + (p-1)(2na + n)$ dan $j = 2na + n - 1$

   that is

   $g(v_i v_j) = ((4na + 2n - a - 1) + (p-1)(2na + n) + 2na + n - 1) - q$

   $= ((4na + 2n - a - 1) + (p-1)(2na + n) + 2na + n - 1)$

   $-(4na + 2n - 2a - 1) + p(2na + n)$

   $= a - 1,$

   the maximum value of $g(v_{i'} v_{j'})$ will be obtained if

   $i = 4na + 2n - a - 1$ and $j = 0$ that is

   $g(v_{i'} v_{j'}) = 4na + 2n - a - 1 + 0 = 4na + 2n - a - 1$

   So that $g(v_i v_j) < g(v_{i'} v_{j'})$, then it is proven that $g(v_i v_j) \neq g(v_{i'} v_{j'})$.

3. It will shown that $g(v_i v_j) \neq g(e)$ where $e \in E(C_{2a+1} \times P_n)$,

   in Theorem 2.1 is shown the minimum value of $g(e)$ where $e \in E(C_{2a+1} \times P_n)$ is a while the maximum value is $4na + 2n - a - 2$. So obtained $g(v_i v_j) < g(e) < g(v_{i'} v_{j'})$. It is proven that $(v_i v_j) \neq g(e)$ where $e \in E(C_{2a+1} \times P_n)$.

Because $g(v_i v_j) \neq g(v_{i'} v_{j'})$ for $\imath > \imath'$, $g(v_i v_j) \neq g(v_{i'} v_{j'})$, and $g(v_i v_j) \neq g(e)$ where $e \in E(C_{2a+1} \times P_n)$, it can be said that each edge of the graph $(C_{2a+1} \times P_n) + \overline{K_p}$ different means that harmonious labeling can be done on the graph so that it can be concluded that graph $(C_{2a+1} \times P_n) + \overline{K_p}$ is a harmonious graph.

■

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